Modelling Financial Risk in the South African Stock Market: An Application of the Generalised Extreme Value Distribution (GEVD)

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ABSTRACT: This study analyses the return characteristics of the South African Industrial Index (J520) and South African Financial Index (J580) using the Generalized Extreme Value Distribution (GEVD) to estimate return levels, Value-at-Risk (VaR) and Expected Shortfall (ES). Results show that losses in the Industrial Index follow a short-tailed negative Weibull distribution, indicating bounded downside risk, while its gains follow a light-tailed Gumbel distribution, suggesting limited but unbounded positive returns. In contrast, losses in the Financial Index exhibit a heavy-tailed Fréchet distribution, pointing to the potential for extreme, unbounded losses during market downturns, whereas its gains are bounded and follow a short-tailed Weibull distribution. These findings highlight the Financial Index's greater vulnerability to extreme losses, especially under volatile conditions, warranting higher risk-based capital allocation. The GEVD model proves to be a robust and flexible tool for modelling extreme return behaviour and enhances portfolio risk management by enabling more accurate estimation of return levels, VaR and ES, thereby supporting better-informed investment decisions under financial stress.

JEL classification: C580, G170.

Keywords: Generalised Extreme Value Distribution, Value at Risk, Expected Shortfall, Extreme Value Theory, Block Maxima Method, Stock Market.

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1 Introduction and motivation

This study applies the Generalised Extreme Value Distribution (GEVD) to analyse financial returns of the South African Industrial Index (J520) and South African Financial Index (J580), addressing the limitations of traditional Normal distribution models. Such distributions often underestimate risk by failing to adequately capture extreme market events, potentially leading to underestimation of losses during periods of financial stress (Embrechts et al., 1997; McNeil et al., 2005; Coles, 2001). In contrast, the GEVD is specifically designed to model heavy tail behaviour, making it a robust tool for assessing extreme gains and losses in financial markets. By focusing on the GEVD, this study aims to enhance tail risk estimation and provide a more accurate framework for understanding financial extremes (Gilli and Këllezi, 2006).

Stock markets around the world experienced a number of crashes between 1980 and 2016 (Rahman et al., 2017). The stock market crashes include the Black Monday (1987-1988), the Asian Financial Crisis (1997-1998), the Global Financial Crisis (GFC) (2007-2008) and the Chinese Financial Crisis (2015-2016). The stock market crashes further highlight the weaknesses of the traditional Normal distribution-based models in estimating extreme market risk. Accurately predicting and hedging against extreme events is difficult in practice and requires appropriate models. Estimating extreme events in the South African industrial and financial markets plays a vital role in an efficient risk appraisal and mitigation against risk exposure.

The increasing risks and uncertainties in the stock market have challenged stock market participants to explore the different theoretical models that can deal with events of an extreme nature. In modelling the behaviour of extreme events that occur in the tails of the return distributions, the Extreme Value Theorem (EVT) method provides reliable models. The ability to forecast extreme risk requires investors and practitioners to use the EVT related models. This study focuses on the modelling of appropriate distributions of extreme returns using the South African Industrial and Financial Indices. The modelling is based on the EVT methods, which contain two fundamental extreme value distributions, viz: the GEVD and Generalised Pareto Distribution (GPD). The GEVD model is preferred in some situations because it circumvents the problems of dependence, which greatly complicates the application of the Generalised Pareto Distribution (GPD) model (Rahman et al., 2017). However, in some situations, the GPD is preferred since it is more efficient at utilising data than the GEVD. The modelling of extreme events relies on the EVT, which provides a sound theoretical framework.

Wainaina and Waititu (2014) concluded that the investors' and practitioners' greatest challenges are in applying risk models that allow for analysing financial returns and forecasting extreme risk. Understanding the expected frequency and magnitude of extreme financial returns requires investors and practitioners to use precise statistical models that would accurately forecast extreme risk in an investment portfolio.

The GEVD model is used to evaluate return levels, VaR and ES more accurately.

The GEVD simplifies statistical implementation, enhances flexibility, and improves the efficiency of extreme value analysis (Coles, 2001). The EVT method provides a set of ready-made models that are used to formulate models that estimate varying risk levels of an extreme nature to quantify and mitigate against financial instability (Embrechts et al., 1997). The findings of this study indicate that the Financial Index exhibits greater risk compared to the Industrial Index, underscoring the need for tailored risk management strategies across sectors. This research supports decision-makers with statistically sound tools for anticipating and managing financial instability.

1.1 Statement of the problem

The inadequacy of statistical parent distributions in accurately capturing extreme risks has led investors and practitioners to adopt more specialised models that better account for extreme tail behaviour. To address this inadequacy, the study proposes to use the GEVD to estimate return levels, VaR and ES of the South African financial and industrial indices. These risk measures are crucial for effectively monitoring and hedging against exposure to extreme risks while maximising potential gains with a high degree of accuracy. This study aims to enhance risk assessment and management in the South African stock market, particularly during periods of financial turbulence.

1.2 Justification of the study

Jiang et al. (2018) concluded that predicting stock market crashes, bank failures and foreign exchange crises is of great interest to investors and practitioners since the events result in financial instability and depletion of asset value. Estimation of return levels, VaR and ES as risk measures assuming statistical parent distributions may lead to underestimation of risk, which can lead to huge unexpected losses as they often fail to capture extreme tail behaviour. According to Alam et al. (2018), the application of the GEVD model is necessary in that it accurately captures the level of extreme risk exposure. Research on the occurrence of financial extremes in the South African Industrial and Financial Indices enables investors to estimate VaR, ES and return levels as measures of risk associated with the indices in advance. The findings will provide quantitative indicators for investors and practitioners who want to invest in the South African stock market, specifically the South African industrial and financial markets.

1.3 Objectives of the study

The objective of the study is to model extreme risk associated with the South African Industrial and Financial Indices using the GEVD model. The specific objectives are to: (i) fit the GEVD model to the South African Industrial and Financial Indices returns (separately for losses and gains), and (ii) estimate return levels (maximum losses/gains),

VaR and ES of South African Industrial and Financial Indices using the GEVD model and compare the riskiness of losses and gains between the two indices.

1.4 Contribution and structure of the study

This study provides empirical evidence that contributes to the literature on the future behaviour of the extreme returns in the South African industrial and financial markets using the GEVD model. Future behaviour is quantified through estimation of return levels and return periods for extreme returns. The 8 and 40 quarterly return periods are used to estimate return levels. This study also estimates VaR and ES. The study differs from other studies since it uses the South African Industrial and Financial Indices returns to model future behaviour of extreme returns. This study is organised as follows: Section 2 gives the literature review; Section 3 describes the research methodology; Section 4 contains the data analysis; and Section 5 gives conclusion and future studies.

2 Literature review

The EVT in finance was advanced by key proponents such as Embrechts et al. (1997) and Coles (2001), who used EVT to model the extreme tails of return distributions and estimate tail-related risk measures. A number of studies have employed the GEVD to analyse extreme events in stock markets, using the Block Maxima (BM) approach to extract maxima data. Some notable contributions in this area include works by Magnou (2017), Makhwiting et al. (2014), Hasan et al. (2012), Allen et al. (2011), Mancini and Trojani (2011), Onour (2009), Neftci (2000), Gilli and Këllezi (2006), Këllezi and Gilli (2000), and McNeil (1998). A few studies are discussed in this section.

Magnou (2017) estimated tail-related risk measures using the GEVD model for the AFAP ASURA pension fund returns in Uruguay. The study used return levels to estimate the portfolio's maximum losses. The MLE method was used to estimate the distribution parameters. The return levels were estimated for return periods of 5, 10, 20, and 50 years and were interpreted as a stress index to help investors mitigate against exposure to extreme risk.

Mwamba et al. (2017) applied EVT to analyse the extreme tail risk linked to daily log return distributions. Their analysis used data from the Standard and Poor's Europe (SPEU), the US Standard and Poor's (S&P 500), the Dow Jones Islamic Market (DJIM), and the Asian Standard and Poor's (SPA S50) covering the period from 1998 to 2015. They employed the POT methods for modelling and their results revealed that the BM method outperformed POT across all four stock indices. Their findings indicated that Islamic stocks exhibited lower risk than the other indices in terms of extreme returns and a higher probability of experiencing extreme gains.

Ramadhani et al. (2017) discussed the GEVD's effectiveness in modelling the maxima distribution. The researchers demonstrated that the GEVD provides a suitable fit regardless of the underlying distribution from which the maxima are derived. Their findings indicated that the maxima would converge to one of three types within the GEVD family: Gumbel, Weibull, or Fréchet, as established by Fisher and Tippett (1928). Their findings also emphasised the significance of block size, noting its direct impact on the shape parameter.

Hussain and Li (2015) analysed the distribution of daily returns for the Shanghai Stock Exchange (SSE) Composite Index from 1991 to 2013. They fitted the GEVD, GPD, and Generalised Logistic Distributions to the dataset. The maxima were selected over various timeframes, from weekly to yearly. Among these, the GEVD yielded the best fit for the maxima series and demonstrated strong performance in modelling extreme upward movements.

Hasan et al. (2012) modelled the extreme tails of share returns behaviour. The data were from the daily closing prices of the Financial Times Stock Exchange (FTSE), Bursa Malaysia Kuala Lumpur Composite Index (KLCI), from 1990 to 2010. The data were modelled using the GEVD. The study employed different block sizes of up to a year, and they all led to similar estimates of the return level and return period as risk measures. The results showed that the yearly maxima distribution better fit the GEVD. The results also revealed that the recorded return level of 20.174% is expected to be exceeded after a return period of 50 years.

Gilli and Këllezi (2006) estimated return levels using the block maxima (BM) approach when selecting extremes. The GEVD was applied for the ten yearly block maxima periods. According to Gilli and Këllezi (2006) the monthly, quarterly and yearly periods are appropriate when avoiding seasonal effects or autocorrelation. The GEVD was used to estimate return levels as risk measures for quarterly return periods of 4, 40, and 400.

Markose and Alentorn (2005) fitted the GEVD to create an option pricing model. The analysis used daily settlement prices of FTSE 100 Index call and put options from 1997 to 2003. The GEVD outperformed the Black-Scholes model, exhibiting a smaller bias as options approached maturity and effectively capturing the tails of the distribution.

Byström (2004) used EVT to analyse the behaviour of extreme tails in financial return distributions. The first dataset comprised the left tail of Sweden Affärsvärlden General Index from January 1980 to November 1999, while the second dataset included the US Dow Jones Industrial Average for the same period. Both datasets consisted of daily closing prices from trading days, and the tails were modelled using the BM and POT methods. The results revealed that both the BM and POT methods, along with time series techniques, resulted in the most effective modelling of the tails.

Longin (2000) analysed both the right and left tails of the S&P 500 Index from 1962 to 1993. The following distributions were used: the GEVD, Normal distribution, Historical Simulation, GARCH process and the EWMA process. The MLE method was used

to estimate the parameters. In the case of the GEVD, weekly, monthly, quarterly and semester blocks were chosen. The results showed that the Fréchet type distribution of the GEVD model offered a better fit for the minima and maxima. A comparison of the VaR estimates of the GEVD was conducted and compared against the rest of the mentioned distributions. It was observed that the GEVD produced superior estimates of VaR at a 99% confidence interval. The Normal distribution-based risk models underestimated VaR and were susceptible to increased model risk error. In contrast, the GEVD reduced this risk as it was flexible due to its makeup comprising the Fréchet, Gumbel and negative Weibull when analysing extreme risk.

The use of EVT, particularly the GEVD, has become a widely accepted approach for modelling extreme financial returns and tail-related risk measures such as return levels, VaR and ES. Foundational works by Embrechts et al. (1997) and Coles (2001) established the theoretical basis for EVT in finance, while empirical studies such as those by Magnou (2017), Hasan et al. (2012), and Gilli and Këllezi (2006) have applied the GEVD using the BM method to capture the severity and frequency of extreme losses. These studies emphasize the importance of appropriate block size selection and confirm the convergence of return maxima to one of the three GEVD families: Gumbel, Weibull, or Fréchet Ramadhani et al. (2017). Research by Longin (2000), Byström (2004), and Mwamba et al. (2017) further demonstrates GEVD's superior performance in modelling heavy-tailed financial data and estimating extreme losses and gains compared to traditional models. Across various markets and indices, the GEVD has consistently been shown to offer robust, flexible modelling of both upside and downside tail risks, thereby enhancing the accuracy of risk assessment, capital allocation, and investment decision-making under stress conditions.

3 Methodology

According to Ocran (2015), the two main methods used to estimate extreme events are the GEVD and GPD models. In this study, the GEVD model with MLE is used for parameter estimation, and its theory is discussed.

3.1 Extreme value distributions

The EVT is a statistical approach that analyses extreme events, tail distributions, and rare occurrences in data Coles (2001). According to Ocran (2015), the GEVD and the GPD are the two main extreme value distributions used to analyse extreme events. The GEVD is the limiting distribution of normalised maxima, whereas the GPD is the limiting distribution of normalised data beyond some high threshold Danielsson (2011); Beirlant et al. (2006); Coles (2001). The GEVD used to analyse extreme financial returns (extreme losses and gains) and forecast the associated risk, is presented in this section.

3.2 Generalised Extreme Value Distribution (GEVD)

The GEVD uses a BM approach to select extreme values, focusing on the distribution of the maximum observation taken from each block. This involves splitting the dataset into mutually exclusive blocks of equal size. The new dataset formed from considering only the largest event from each block is assumed to follow the GEVD. The PDF of the GEVD is given in equation (1):

$$h_{\gamma,\mu,\sigma}(x) = \begin{cases} \dfrac{1}{\sigma}\left(1+\dfrac{\gamma(x-\mu)}{\sigma}\right)^{-1-\frac{1}{\gamma}} \exp\left\{-\left(1+\dfrac{\gamma(x-\mu)}{\sigma}\right)^{-1/\gamma}\right\} & \gamma \neq 0, \\ \dfrac{1}{\sigma}\exp\left\{-\dfrac{x-\mu}{\sigma} - \exp\left(-\dfrac{x-\mu}{\sigma}\right)\right\} & \gamma = 0. \end{cases} \tag{1}$$

The CDF of the GEVD is given in equation (2):

$$H_{\gamma,\mu,\sigma}(x) = \begin{cases} \exp\left\{-\left(1+\gamma\,\dfrac{x-\mu}{\sigma}\right)^{-1/\gamma}\right\} & \gamma \neq 0, \\ \exp\left\{-\exp\left(-\dfrac{x-\mu}{\sigma}\right)\right\} & \gamma = 0, \end{cases} \quad \text{for } 1+\gamma\,\frac{x-\mu}{\sigma} > 0, \tag{2}$$

where x is the return variable, μ is the location parameter, σ is the scale parameter, and γ is the shape parameter. If $\gamma > 0$ it is a Fréchet class distribution; if $\gamma = 0$ it is a Gumbel class distribution; and if $\gamma < 0$ it is a negative Weibull class distribution.

The GEVD model forecasts the return levels, VaR and ES. The MLE is used in this study to estimate the shape, scale and location parameters Coles (2001).

3.2.1 Parameter estimation for the GEVD using MLE

The parameters of the GEVD are estimated using the log-likelihood function. The log-likelihood function (equation 3) is maximised by differentiating with respect to each of the parameters to obtain the parameters μ, σ and γ using the MLE method Coles (2001). The MLE method is widely used because of its reliability and simplicity Lazoglou and Anagnostopoulou (2017).

The log-likelihood function of the GEVD is:

$$\ell(\mu,\sigma,\gamma \mid x) = -n\ln\sigma - \left(1+\frac{1}{\gamma}\right)\sum_{i=1}^{n}\ln\left(1+\gamma\,\frac{x_i-\mu}{\sigma}\right) - \sum_{i=1}^{n}\left(1+\gamma\,\frac{x_i-\mu}{\sigma}\right)^{-1/\gamma}, \tag{3}$$

where x is the return variable, μ is the location parameter, σ is the scale parameter, and γ is the shape parameter.

The log-likelihood function is differentiated with respect to μ, σ, and γ, equated to zero, and can be solved using numerical methods.

3.2.2 Selecting the block minima/maxima

Each block has a maximum or minimum, which shares a common distribution with other blocks and converges to a GEVD model (Këllezi and Gilli, 2000). In the present work the quarterly block intervals are applied to monthly data, resulting in 91 right and left tail blocks. The 91 maximum and minimum points are sufficient for GEVD model analysis.

3.2.3 Modelling minima/maxima values

Let $X_1, X_2, \ldots, X_n$ be a sequence of i.i.d values. The duality principle between the distributions of the minima and maxima is utilised to fit the distribution to the maximum losses (minima): For a sequence $X_1, X_2, \ldots, X_n$ of i.i.d. returns, one can derive the minima (loss) distribution by examining the distribution of maximum negative returns (large losses). Let $M_n = \min\{X_1, X_2, \ldots, X_n\}$ be the minimum for n-observations. The block minima problem becomes:

$$\widehat{M_n} = \max\{-X_1, -X_2, \ldots, -X_n\},$$

where X_i for $i = 1, 2, \ldots, n$ are equity returns.

The block maxima problem becomes:

$$M_n = \max\{X_1, X_2, \ldots, X_n\},$$

where M_n is the maximum over an n-observations period. The transformation of minima to maxima is a powerful technique that leverages the underlying relationships between extreme values in probability distributions.

3.2.4 Return levels and return periods for the GEVD

The GEVD model is used to estimate return levels (alternative measures of risk) and their interval estimates. The parameters are estimated using the MLE method. The return levels (RL) for the GEVD model are given by equation (4):

$$RL = \begin{cases} \mu - \frac{\sigma}{\gamma}\left[1 - \big(-\log(1-p)\big)^{-\gamma}\right] & \text{for } \gamma \neq 0, \\ \mu - \sigma \log\big(-\log(1-p)\big) & \text{for } \gamma = 0, \end{cases} \tag{4}$$

where x is the return variable, μ is the location parameter, σ is the scale parameter, and γ is the shape parameter.

The return period T is the average time between events that exceed the return level (RL). It is inversely related to the exceedance probability p and is given by equation (5):

$$p = \frac{1}{T}. \tag{5}$$

4 Data analysis and discussion

This study applied the GEVD model to the monthly returns of the South African Industrial Index (J520) and South African Financial Index (J580) over the period 1995–2018 to estimate and compare the extreme financial risk of the two indices.

4.1 Software used and research data

The following packages were used for data analysis in the R programming environment: actuar, QRM, Mass, evir, eva, evd, fExtremes, extRemes, and VARES.

This study makes use of secondary data of the South African stock market, specifically the monthly South African Industrial Index (J520) and South African Financial Index (J580), which were obtained, with permission, from the website Iress Expert (https://expert.inetbfa.com). The analysis involved the J520 and J580 returns (gains and losses) distributions, spanning the years 1995-2018. These indices are calculated from values of stocks in the industrial and financial sector companies listed on the South African stock market, and represent the performance of those specific industries within the stock market.

4.2 Analysis of losses and gains

Losses and gains are analysed separately. The minima and maxima were fitted individually to the GEVD. The Block Maxima (BM) approach is used to extract quarterly minima and maxima from the South African Industrial and Financial Indices returns.

Maxima for losses and gains are fitted to the GEVD, and the resulting models are used to estimate parameters, return levels, VaR, and ES as measures of tail-related risk. The monthly South African Industrial and Financial Indices are divided into quarterly blocks of three months each, resulting in 91 blocks for each index. The quarterly block maxima diagrams for the losses of the two indices are presented. The series is inverted by multiplying with –1 to expose maximum losses as positive values. The maxima values for the South African Industrial and Financial Indices returns were extracted, fitted to the GEVD model, and are shown in Figures 1 and 2, respectively. For each block, the largest value is taken as the maxima.

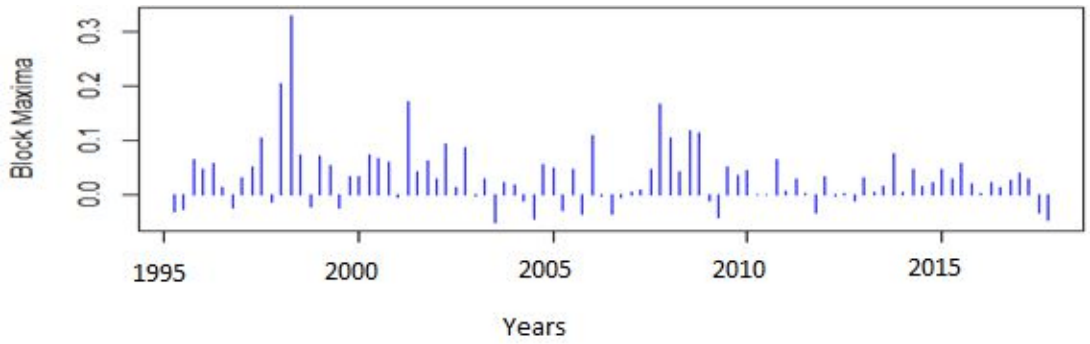
4.3 Fitting the Generalised Extreme Value Distribution (GEVD) model

The GEVD model is fitted separately to the maxima returns of the losses and gains.

4.3.1 Model diagnostics for the maxima

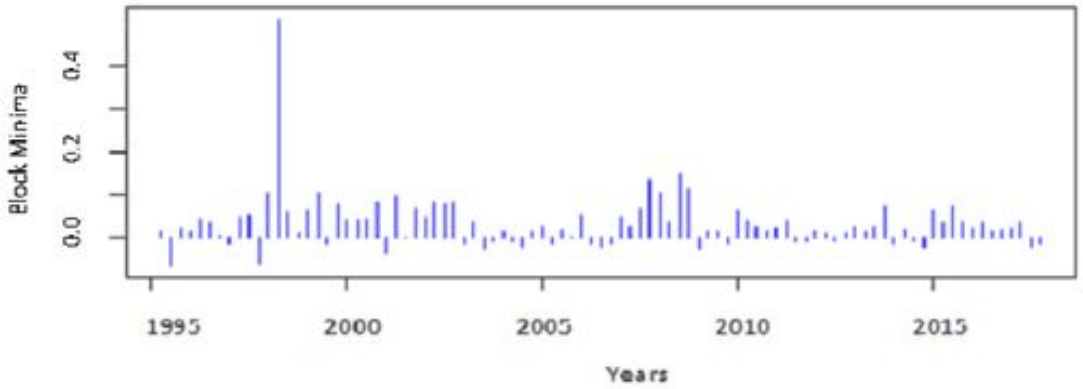
Figures 3 and 4 show GEVD diagnostic plots for quarterly block maxima of losses in the South African Industrial and Financial Indices.

Figure 1: Block maxima (n = 91) extracted from the loss returns of the South African Industrial Index



Notes: Figure presents the 91 maximum negative return values (losses), identified using the block maxima method, to model extreme losses in the South African Industrial Index.

Figure 2: Block maxima (n = 91) extracted from the loss returns of the South African Financial Index



Notes: Figure presents the 91 maximum negative return values (losses), obtained using the block maxima method, for modeling extreme losses in the South African Financial Index.

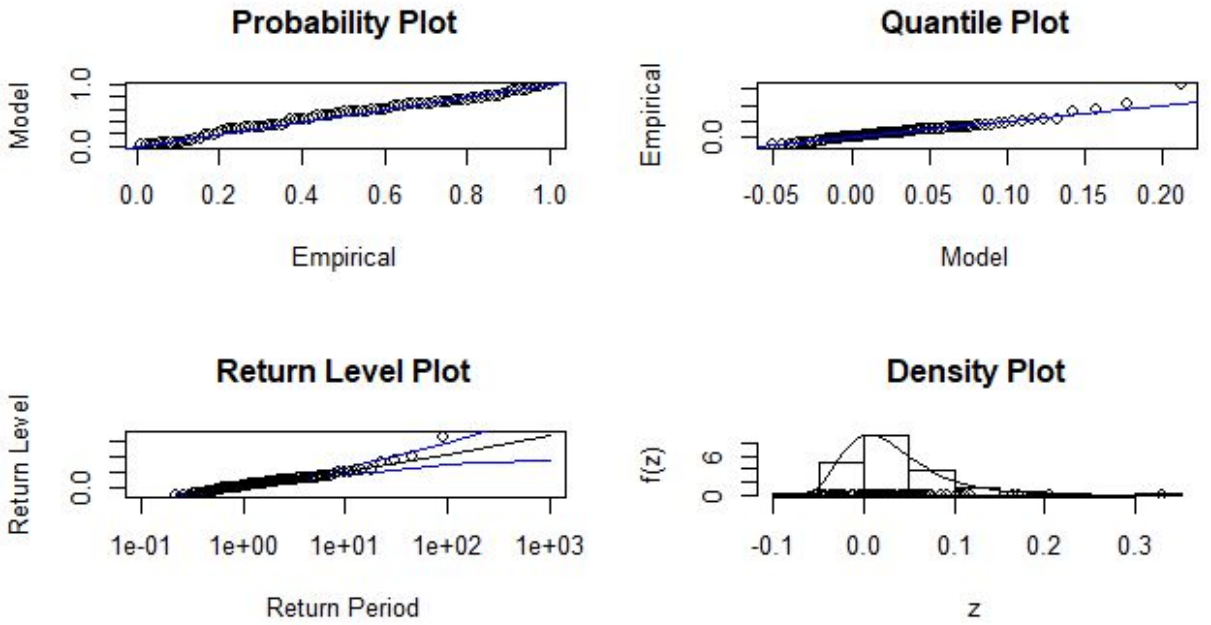
The diagnostic plots in Figures 3 and 4 assess the fit of the GEVD model to the quarterly minima of losses for the South African Industrial and Financial Indices. Data points in the Q–Q plots do not deviate significantly from the straight line, and the density plots confirm the conclusions obtained from the probability plots that the GEVD model provides a good fit for the tails of the distributions. The assumptions for fitting the GEVD to the data appear reasonable, and the diagnostic plots support the suitability of the model for the South African Industrial Index (J520) and Financial Index (J580).

4.3.2 Estimation of parameters

Fitting the GEVD model to the quarterly losses and gains data allows for estimation of the parameters using the MLE method.

For the losses in Table 1, the shape parameter (γ) is -0.2916. Since $\gamma < 0$, the losses

Figure 3: Model diagnostics for quarterly block maxima of loss returns in the South African Industrial Index



Notes: Figure presents diagnostic plots used to assess the fit of the extreme value model applied to quarterly block maxima extracted from the negative return series of the South African Industrial Index.

follow a Weibull class distribution, implying they are upper-bounded. For the gains in Table 1, the shape parameter (γ) is 0.0536. Since $\gamma > 0$, the gains follow the heavy-tailed Fréchet class distribution. The parameters in Table 1 indicate that the South African Industrial Index gains are riskier than its losses.

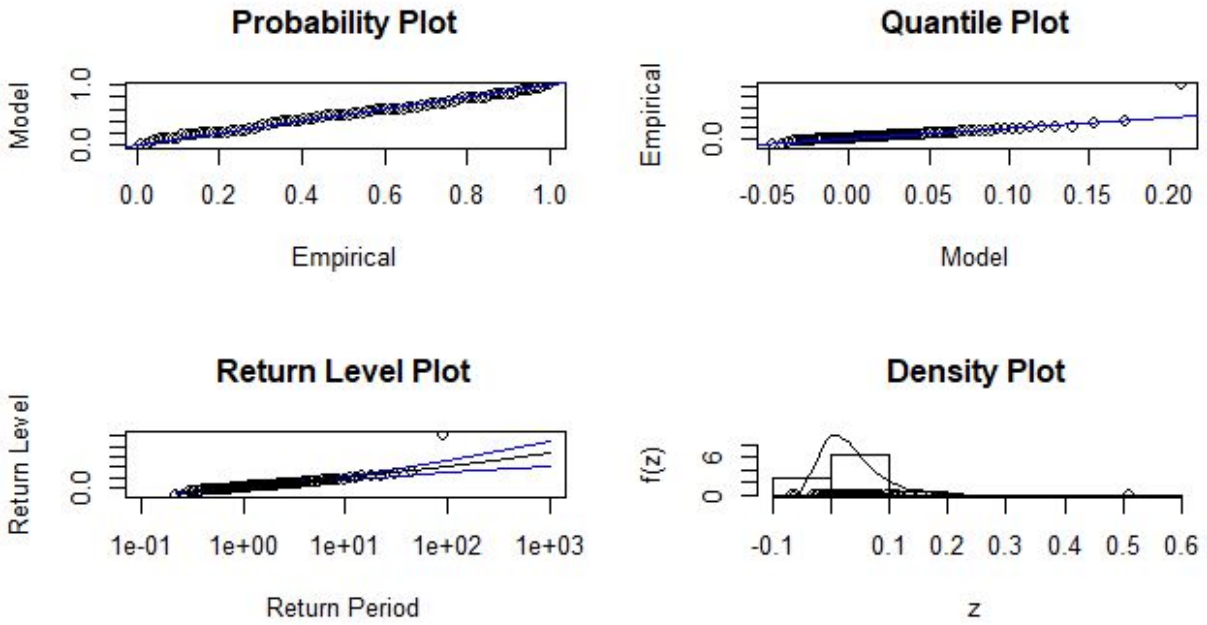
Table 1: Summary statistics of losses and gains in the South African Industrial Index return series

Parameter	Lower Bound	Point Estimate	Upper Bound
Parameters for maxima values of the losses (95% confidence level)			
Shape parameter, γ	−0.4185	−0.2916	−0.1647
Scale parameter, σ	0.0331	0.0392	0.0453
Location parameter, μ	0.0295	0.0384	0.0472
Parameters for maxima values of the gains (95% confidence level)			
Shape parameter, γ	0.0016	0.0536	0.1056
Scale parameter, σ	0.0374	0.0400	0.0429
Location parameter, μ	0.0042	0.0077	0.0112

Notes: Table presents key parameters characterising the distribution of positive (gains) and negative (losses) returns in the South African Industrial Index.

For the losses in Table 2, the shape parameter (γ) is 0.0583, indicating that the South African Financial Index returns follow a Fréchet class distribution. For the gains, the

Figure 4: Model diagnostics for quarterly block maxima of loss returns in the South African Financial Index



Notes: Figure presents diagnostic plots evaluating the fit of the extreme value model applied to quarterly block maxima derived from the negative return series of the South African Financial Index.

shape parameter (γ) is -0.0674, indicating that they follow a Weibull class distribution and are upper-bounded. Overall, the parameters in Table 2 indicate that losses in the South African Financial Index are riskier than gains. This contrasts with the conclusion for the South African Industrial Index.

Table 2: Summary statistics of losses and gains in the South African Financial Index return series

Parameter	Lower Bound	Point Estimate	Upper Bound
Lower tail (Losses) – 95% confidence level			
Shape parameter, γ	0.0005	0.0583	0.1161
Scale parameter, σ	0.0346	0.038 48	0.0424
Location parameter, μ	0.0026	0.0082	0.0138
Gains – 95% confidence level			
Shape parameter, γ	-0.1350	-0.0674	-0.0001
Scale parameter, σ	0.0331	0.0362	0.0393
Location parameter, μ	0.0284	0.0328	0.0371

Notes: Table presents key parameters characterising the distribution of positive (gains) and negative (losses) returns in the South African Financial Index.

4.3.3 Estimation of return levels

Given the parameters, the return levels are estimated as tail-related risk measures Gilli and Këllezi (2006). In Table 3, for the South African Industrial Index losses, the return level for a 2-year return period (8 quarters) is 0.0981. This indicates that, on average, the expected return of 0.0981 is exceeded once every 2 years at a 95% confidence level. For a 10-year return period (40 quarters), the return level is 0.1268, meaning it is expected to be exceeded once every 10 years at a 95% confidence level. Return levels for other periods can be interpreted similarly.

Table 3: Estimated return levels for the South African Industrial and Financial Indices

Return Period (Quarters)	8 (2 years)	40 (10 years)
South African Industrial Index		
Losses: return levels	0.0981	0.1268
Gains: return levels	0.0928	0.1703
South African Financial Index		
Losses: return levels	0.0904	0.1660
Gains: return levels	0.1010	0.1506

Notes: Table presents return levels corresponding to various return periods, providing insight into the expected magnitude of extreme positive or negative returns over time.

The return level estimates for the South African Financial Index losses are greater than those for the South African Industrial Index losses. This means that the South African Financial Index losses are riskier than the South African Industrial Index losses. In contrast, the return level estimates for the South African Financial Index gains are less than those for the South African Industrial Index gains. This means that the South African Financial Index gains are less risky than the South African Industrial Index gains.

4.3.4 Estimation of VaR and ES

In most practical situations, the GEVD model is used to estimate return levels, although in some situations it is used to estimate VaR and ES. To estimate the VaR and ES of financial returns using the BM method, the financial return data were divided into quarterly non-overlapping blocks, and 91 blocks were obtained for each dataset. The maximum return within each block was identified to create a new dataset of block maxima. The GEVD is fitted to each dataset, and the parameters, namely the location, scale, and shape parameters, are estimated using the MLE method. The VaR and ES at a specified confidence level are then estimated for the losses and gains.

In Table 4, for the South African Industrial Index losses at a 95% confidence level, the VaR and ES estimates are 0.1163 and 0.1642, respectively. A VaR of 0.1163 means that there is a 95% chance that the loss will not exceed 11.63% of the portfolio value

over a period of one month. The ES of 0.1642 indicates that if the loss exceeds 11.63%, the average loss in those worst 5% scenarios will be 16.42% of the portfolio value. This implies that losses are not expected to exceed 0.1163; however, if they do, the average loss will be 0.1642. This interpretation applies to all other estimates, including those for the South African Financial Index.

Table 4: Univariate risk measure estimates for gains and losses in the South African Industrial and Financial Indices

	South African Industrial Index		South African Financial Index	
Confidence level	VaR	ES	VaR	ES
Losses				
0.950	0.1163	0.1642	0.1330	0.1778
0.990	0.1377	0.1887	0.2112	0.2426
0.995	0.1442	0.1956	0.2469	0.2783
Gains				
0.950	0.1365	0.1614	0.1302	0.1759
0.990	0.2164	0.2470	0.1759	0.2258
0.995	0.2527	0.2843	0.1940	0.2445

Notes: Table reports univariate risk measures—including Value at Risk (VaR) and Expected Shortfall (ES)—for both positive (gains) and negative (losses) return distributions of the South African Industrial and Financial Indices.

In Table 4, the univariate VaR and ES estimates for the returns of the South African Industrial and Financial Indices are provided. The VaR and ES values for the South African Financial Index losses are higher than those for the South African Industrial Index losses, indicating that the Financial Index carries greater risk in terms of potential losses. This affirms the notion that the Financial sector is riskier than the Industrial sector (McNeil et al., 2005; Embrechts et al., 1997; Schwert, 1989). In contrast, the VaR and ES estimates for gains in the South African Financial Index are lower than those for the South African Industrial Index gains, suggesting that the Financial Index is less risky in terms of potential gains. These findings can help investors and practitioners determine more appropriate, risk-adjusted capital requirements for the two indices. Based on the higher risk associated with the South African Financial Index, the recommendation is to increase the capital requirements for this Index. In summary, the parameter estimates, return levels, and VaR/ES estimates indicate that, regardless of the estimation method used, the Financial Index shows significantly greater potential for extreme losses.

4.3.5 Discussion

The findings of this study align with and extend the existing literature on the application of the GEVD in modelling financial extremes. Consistent with the theoretical expectations of Ramadhani et al. (2017) and empirical results by Longin (2000), the study confirms that return maxima converge to one of the three GEVD types. Specifically, losses in

the South African Industrial Index follow a short-tailed Weibull distribution, indicating bounded downside risk, while losses in the Financial Index follow a heavy-tailed Fréchet distribution, suggesting the presence of potentially extreme and unbounded losses. This contrast highlights the Financial Index's greater susceptibility to extreme negative returns, in line with the conclusions of Byström (2004) and Mwamba et al. (2017) for global stock indices. In addition, the identification of Gumbel-distributed gains in the Industrial Index supports findings by Hussain and Li (2015), who noted the GEVD's suitability for modelling light-tailed positive returns. The accurate estimation of return levels, VaR, and ES using the BM method reinforces the methodological strengths highlighted by Gilli and Këllezi (2006) and Hasan et al. (2012), affirming the GEVD's superiority over traditional risk models under extreme market conditions.

Beyond methodological validation, this study offers new insight by applying the GEVD to South African financial data, uncovering market-specific risk asymmetries with direct implications for risk-based decision-making. The results demonstrate that during periods of financial distress, the South African Financial Index is significantly riskier than the Industrial Index due to its higher potential for extreme losses. This finding is consistent with general expectations in the financial literature that the financial sector, owing to its leverage exposure and sensitivity to macroeconomic shocks, tends to be more volatile than the Industrial sector (McNeil et al., 2005; Embrechts et al., 1997; Schwert, 1989). In contrast, the Industrial Index, comprising firms in manufacturing, services, and production, reflects more stable and predictable return behaviour. These insights are valuable for investors, risk managers, and policymakers, as they emphasize the importance of tail-risk modelling in portfolio construction, capital allocation, and regulatory stress testing, particularly in emerging markets where sector-specific vulnerabilities may be more pronounced.

5 Conclusion and areas of future studies

5.1 Conclusion

This study analysed the returns of the South African Industrial and Financial Indices and estimated return levels, VaR, and ES using the GEVD. The study revealed that losses in the South African Industrial Index follow a short-tailed negative Weibull class distribution, implying that losses are bounded for this Index. The gains of the South African Industrial Index follow a light-tailed Gumbel class distribution. In contrast, losses in the South African Financial Index follow a heavy-tailed Fréchet class distribution, implying the potential for extreme and unbounded losses. The gains of the South African Financial Index follow a short-tailed negative Weibull class distribution, indicating that gains are bounded. These results suggest that higher risk-based capital requirements should be allocated to the South African Financial Index during periods of market turbulence to help mitigate unpredictable losses. Overall, the GEVD model provides a flexible and

effective framework for estimating return levels, VaR, and ES, thereby supporting efficient portfolio risk management aimed at minimising exposure to extreme risk while maximising potential gains.

5.2 Areas of further studies

Modelling extreme values using the GEVD may result in data loss in extreme value analysis if one block contains more extreme observations than another (Coles, 2001). Gilli and Këllezi (2006) concluded that the GPD may be a more suitable model, as it makes more efficient use of available data. The Balkema–de Haan (Balkema and de Haan, 1974) and Pickands (Pickands, 1975) Theorem extends the Fisher–Tippett (Fisher and Tippett, 1928) and Gnedenko (Gnedenko, 1943) Theorem to order statistics above a chosen threshold, demonstrating that they converge to a GPD as the threshold increases. The practical implication is that the GPD is often preferable because it utilises a larger proportion of the data. Therefore, future research will investigate and estimate univariate return levels, VaR, and ES using the GPD.

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References

Alam, M. A., Emura, K., Farnham, C., and Yuan, J. (2018). Best-fit probability distributions and return periods for maximum monthly rainfall in Bangladesh. *Climate*, 6(1):9.

Allen, D. E., Singh, A. K., and Powell, R. J. (2011). *Extreme Market Risk: An Extreme Value Theory Approach*. Edith Cowan University Publication.

Balkema, A. A. and de Haan, L. (1974). Residual life time at great age. *The Annals of Probability*, 2(5):792–804.

Beirlant, J., Goegebeur, Y., Segers, J., and Teugels, J. L. (2006). *Statistics of extremes: Theory and applications*. John Wiley & Sons, Chichester, UK.

Byström, H. N. E. (2004). Managing extreme risks in tranquil and volatile markets using conditional extreme value theory. *International Review of Financial Analysis*, 13(2):133–152.

Coles, S. (2001). *An Introduction to Statistical Modeling of Extreme Values*. Springer, Bristol, UK, 3rd edition.

Danielsson, J. (2011). *Financial risk forecasting: The theory and practice of forecasting market risk with implementation in R and Matlab*. John Wiley & Sons.

Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modelling Extremal Events for Insurance and Finance*. Springer-Verlag.

Fisher, R. and Tippett, L. (1928). Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Mathematical Proceedings of the Cambridge Philosophical Society*, 24(2):180–190.

Gilli, M. and Këllezi, E. (2006). An application of extreme value theory for measuring financial risk. *Computational Economics*, 27(2–3):207–228.

Gnedenko, B. V. (1943). Sur la distribution limite du terme maximum d'une série aléatoire. *Annals of Mathematics*, 44:423–453.

Hasan, H., Radi, N. F. A., and Kassim, S. (2012). Modeling the distribution of extreme share return in Malaysia using generalized extreme value (GEV) distribution. In *AIP Conference Proceedings*, volume 1450, pages 82–89.

Hussain, S. I. and Li, S. (2015). Modelling international diversification between the Chinese Stock Market and others. In *Financial Markets & Corporate Governance Conference*. The University of Western Australia.

Jiang, Z. Q., Wang, G. J., Canabarro, A., Podobnik, B., Xie, C., Stanley, H. E., and Zhou, W. X. (2018). Short term prediction of extreme returns based on the recurrence interval analysis. *Quantitative Finance*, 18(3):353–370.

Këllezi, E. and Gilli, M. (2000). Extreme value theory for tail-related risk measures. Preprint, University of Geneva.

Kotz, S. and Nadarajah, S. (2000). *Extreme value distributions: Theory and applications*. World Scientific Publishers.

Lazoglou, G. and Anagnostopoulou, C. (2017). An overview of statistical methods for studying the extreme rainfalls in Mediterranean. In *Proceedings*, volume 1, page 681.

Longin, F. M. (2000). From value at risk to stress testing: The extreme value approach. *Journal of Banking and Finance*, 24(7):1097–1130.

Magnou, G. (2017). An application of extreme value theory for measuring financial risk in the Uruguayan Pension Fund. *Compendium: Cuadernos de Economía y Administración*, 4(7).

Makhwiting, M. R., Sigauke, C., and Lesaoana, M. (2014). Modelling tail behavior of returns using the generalized extreme value distribution. *Economics, Management and Financial Markets*, 9(1):41–52.

Mancini, L. and Trojani, F. (2011). Robust value at risk prediction. *Journal of Financial Econometrics*, 9(2):281–313.

Markose, S. and Alentorn, A. (2005). Option pricing and the implied tail index with the generalized extreme value (GEV) distribution. Computing in Economics and Finance Working Paper 397, Society for Computational Economics.

McNeil, A. J. (1998). Calculating quantile risk measures for financial return series using extreme value theory. Technical report, Department of Mathematics, ETH Zurich.

McNeil, A. J., Frey, R., and Embrechts, P. (2005). *Quantitative Risk Management: Concepts, Techniques and Tools*. Princeton University Press, Princeton, NJ.

Mwamba, J. W. M., Hammoudeh, S., and Gupta, R. (2017). Financial tail risks in conventional and Islamic stock markets: A comparative analysis. *Pacific-Basin Finance Journal*, 42:60–82.

Neftci, S. N. (2000). Value at risk calculations, extreme events, and tail estimation. *The Journal of Derivatives*, 7(3):23–37.

Ocran, E. (2015). Extreme value modelling of the water levels of the Akosombo Dam. Mphil thesis, University of Ghana, Legon.

Onour, I. (2009). Extreme risk and fat-tails distribution model: Empirical analysis. MPRA Paper 17736, University Library of Munich, Munich Personal RePEc Archive.

Pickands, J. (1975). Statistical inference using extreme order statistics. *The Annals of Statistics*, 3(1):119–131.

Rahman, M. T., Hossain, S. Z., and Habibullah, M. (2017). Stock market crash in Bangladesh: The moneymaking psychology of domestic investors. *American Journal of Theoretical and Applied Business*, 3(3):43–53.

Ramadhani, F. A., Nurrohmah, S., and Novita, M. (2017). Extreme value theory (EVT) application on estimating the distribution of maxima. In *AIP Conference Proceedings*, volume 1862, page 030156.

Schwert, G. W. (1989). Why does stock market volatility change over time? *The Journal of Finance*, 44(5):1115–1153.

Wainaina, H. W. and Waititu, A. G. (2014). Modeling insurance returns with extreme value theory (a case study for Kenya's fire industrial insurance class of business). *Mathematical Theory and Modeling*, 4(2):49–55.