Degree of (In)Efficiency in the Stock Market: Do Price-Earnings Ratios Matter?

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ABSTRACT: Employing an innovative approach based on fractional integration, this study examines the potential heterogeneity in long memory (persistence) behavior among stocks of two groups of companies listed on stock markets: those with higher price-earnings (P/E) ratios and those with lower P/E ratios. This empirical investigation offers a novel contribution to the ongoing economic literature on market efficiency and long memory dynamics, in the period spanning January 2016 to December 2022, based on Brazil's daily stock prices. The fractionally integrated parameter is used to check long memory and, for both returns and volatility, the results reveal that P/E ratios alone do not significantly influence the degree of persistence, as both groups display similar long memory patterns. Notably, the occurrence of persistent volatility is more attributable to external shocks, such as the COVID-19 crisis, than to valuation metrics, such as the P/E ratio. Furthermore, the stock market's degree of (in)efficiency is time-varying and exhibits mean reversion (i.e., it is transient), indicating transitory inefficiencies.

JEL classification: C22, G10, G15.

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1 Introduction

According to Woo et al. (2020), several anomalies can affect the financial market and are contrary to the Efficient Market Hypothesis (EMH), such as the price-earnings ratio effect, winner-loser effect, reversal effect, momentum effect, calendar anomalies (e.g., January effect, weekend effect, and reverse weekend effect), book-to-market (BTM) effect, value anomaly, size effect, disposition effect, equity premium puzzle, herd effect, ostrich effect, and bubbles.

As stated by Lakonishok et al. (1994), researchers and investment professionals claim that value strategies outperform the market. The idea is to buy stocks with low prices relative to earnings, dividends, historical prices, book assets, or other measures of value. Value strategies have gained prominence in this context. For example, Basu (1977), Jaffe et al. (1989), and Chan et al. (1991) demonstrate that stocks with low price-earnings (P/E) ratios deliver higher returns (P/E ratio is an indicator of the future investment performance of a stock); low P/E stocks will tend to outperform high P/E stocks.

In contrast, Daniel and Titman (1997) show that anomalies such as book-to-market (BM) and size only represent investor preferences and do not determine stock returns. Fama (1991) states that, in the semi-strong form, referred to as event studies, certain events can cause temporary distortions in asset prices. However, these prices eventually return to an equilibrium state, reflecting all publicly available information. According to Fama (1998), even in the presence of anomalies and paradoxes, the EMH still holds, as the anomalies documented in several studies are not persistent and disappear when the model, sample, or data frequencies change. To Fama (op. cit.), the finance literature, including behavioral finance, seems to produce many long-term return anomalies. However, consistent with the EMH, which posits that anomalies are the result of randomness (to use a fundamental term from Taleb, 2008), the apparent overreaction of stock prices to information is as common as an underreaction. Additionally, the post-event continuation of pre-event abnormal returns is almost as frequent as the post-event reversal. More importantly, the long-term return anomalies are fragile and tend to disappear.

As highlighted by Lekhal and El Oubani (2020), in addition to the methodological problems noted by Fama (1998), the controversial conclusions can be attributed to the fact that the arguments of both the EMH and behavioral finance are partially valid. From this perspective, alternative theories of market dynamics for EMHs are emerging, as described by Hull and McGroarty (2014). These new approaches see efficiency as something the market tends toward, rather than a state that always maintains itself automatically. For instance, it is possible to cite the Fractal Market Hypothesis (FMH) (Peters, 1991, 1994), and more recently, the Adaptive Market Hypothesis (AMH) proposed by Lo (2004, 2005), which combines behavioral finance concepts with the dynamics of evolution. Unlike EMH, AMH allows for the possibility of serial dependency, at least in the short term. As described by Lo (2004), it seems that AMH can reconcile EMH with all its behavioral alternatives.

The concept of long memory is important and applicable for analyzing the existence of efficiency in the financial market. In finance, this concept gained prominence with Lo (1991), who argued that the presence of long memory in asset returns would distort the market from the Random Walk Hypothesis (RWH). It is noteworthy that the time series with long-term memory exhibit an unusually high degree of persistence such that observations in the remote past are non-trivially correlated with observations in the distant future, even when the time interval between the two observations increases. Moreover, the presence of long memory components in asset returns has important implications for many paradigms used in modern financial economics. For more details, see Lo (1991), Sims (1984), Maheswaran (1990), and Maheswaran and Sims (1992).

In this context, employing an innovative approach based on fractional integration, this study aims to verify whether the stock returns and volatilities of a sample of Brazilian companies listed on the São Paulo Stock Exchange Index (IBOVESPA) exhibit long-memory behavior (persistence) and to examine the potential heterogeneity in such a behavior between two groups: companies with high and low price-earnings (P/E) ratios. The period adopted was from January 1, 2016, to December 31, 2022. In more detail, the idea is to analyze whether, considering the P/E ratio of 2019, that is, before the COVID-19 pandemic, the stock returns and volatility for both groups of companies behaved similarly or differently after the start of the pandemic in terms of long-range dependence. Two hypotheses are formulated: (i) behavior varies over time, especially during periods of turbulence, such as the COVID-19 pandemic, and (ii) there is similar behavior in both groups of companies, indicating that the P/E ratio alone does not lead to persistent (long-memory behavior) heterogeneity of returns and volatility.

In this study, long-range dependency was investigated using the fractionally integrated parameter (d). In the first stage, for all estimates, the Geweke and Porter-Hudak (GPH) estimator (Geweke and Porter-Hudak, 1983) was adopted. To ensure robustness, the Exact Local Whittle (ELW; Shimotsu and Phillips, 2005) estimator was also used. In addition, because the estimated fractionally integrated parameter can vary over time, a rolling estimation is adopted to capture the time variation of $\hat{d}$.

The main contributions of this study are as follows. First, a long memory analysis was conducted for the stocks of two groups: companies with higher P/E ratios and companies with lower P/E ratios. Second, this study uses recent data that includes the COVID-19 pandemic. Third, the degree of persistence (long memory) was estimated for returns and volatility. Fourth, the estimates were obtained using a rolling estimation. To the best of our knowledge, there are no similar studies regarding the Brazilian stock market, so this study can be regarded as a novel contribution to the ongoing economic literature on market efficiency and long memory dynamics. Such research is of great importance because it can assist in the future decisions of economic agents (e.g., investors, companies, and policymakers).

The remainder of this paper is organized as follows. In addition to this introduction, Section 2 provides a literature review. Section 3 presents the data and methodology used. In Section 4, the results and a discussion are presented. Finally, the conclusion is presented in Section 5.

2 Literature review

This section provides a literature review of studies that aim to verify the efficiency of financial markets, highlighting the presence or absence of long-memory behavior (in asset returns and asset price volatility), some of which are specific to the Brazilian economy and others to the COVID-19 pandemic. A few studies specifically address the P/E ratio; however, research comparing two groups of companies, as in this study, was not found. This is the main contribution of the present study to empirical financial literature.

The available empirical evidence for long-range dependence is mixed. Long memory evidence is found by Mandelbrot (1972), Greene and Fielitz (1977), Booth et al. (1982), Abbritti et al. (2016), Caporale et al. (2019), among others. Conversely, the following works did not find long-memory behavior: Lo (1991), Jacobsen (1995), Crato and Ray (2000), Malkiel (2003), Serletis and Rosenberg (2007), Lu and Perron (2010). This discrepancy may be because the degree of time-series persistence can vary over time, as presented by Corazza and Malliaris (2002), Glenn (2007), and Bennett and Gartenberg (2016), due to structural breaks (Charfeddine and Guégan, 2012), or even because of the models and statistical methods adopted (Fama, 1998).

In this context, it is important to note that there is also a debate regarding the presence of long-range dependence for returns and volatility. According to Engle (1982) and Bollerslev (1986), the volatility of financial returns can present a strong autocorrelation structure, while returns show no memory and exhibit random walk behavior. For example, Crato and Ray (2000) examined the memory of future returns using a modified version of the R/S statistic and the GPH estimator of the long-memory parameter. The results demonstrated no long-memory behavior in future returns. However, the findings reveal overwhelming evidence of persistence for volatility (squared log returns), consistent with Ding et al. (1993), Bollerslev and Mikkelsen (1996), Baillie et al. (1996), and Breidt et al. (1998). According to Bhattacharya et al. (2018), there is consensus among the financial community that long memory is a characteristic of asset price volatility, which does not occur for asset returns.

In the case of the Brazilian stock market, studies corroborate EMH and others contradicting it. For example, Resende and Teixeira (2002) found short-memory behavior for IBOVESPA returns, both before and after Real Stabilization (Real Plan). Costa and Vasconcelos (2003) revealed that the structural reforms initiated after 1990 (the Collor Plan in the early 1990s and Real Plan in 1994) led to a more efficient stock market in Brazil. The results of Cavalcante and Assaf (2004) show significant long-memory behavior in volatility measures, while there is little evidence of long memory in the returns of the Brazilian stock market. Cajueiro and Tabak (2004), studying the stock returns of Latin American and Asian countries, suggested that emerging markets are becoming more efficient, except for Brazil, the Philippines, and Thailand.

Recent studies have focused on the COVID-19 pandemic. Vera-Valdés (2022), for instance, analyzed the long-term effects of COVID-19 on the Chicago Board Options Exchange (CBOE) Volatility Index (VIX) and observed variances for several international markets. The results show that volatility measures for most countries experienced an increase in the degree of memory following the pandemic. Additionally, several volatility measures became nonstationary, signaling the start of a period with higher and more persistent financial volatility. In the case of the Brazilian stock market, Monte (2023) demonstrated strong evidence of long-range dependence in the CBOE Brazil ETF volatility index, especially during the pandemic when the level of persistence increased substantially. In addition, dos Santos Maciel (2023) provided evidence that there was multifractality before and after COVID-19, rejecting the RWH.

Finally, regarding the P/E ratio, Basu (1977) empirically verifies whether the investment performance of common stocks is related to their P/E ratios. The data sample represents over 1,400 industrial firms that traded on the NYSE between September 1956 and August 1971. According to the results, P/E ratio information is not "fully reflected" in stock prices as rapidly as postulated by the semi-strong efficient market hypothesis. Instead, imbalances appeared to persist in the capital markets during the study period. Campbell and Shiller (1988) find that initial P/E ratios explain as much as 40 percent of the variance in future returns. Other authors, such as Nicholson (1960) and Ball (1978), reached similar conclusions. At the Brazilian level, Amorim and Camargos (2021), investigating the period from December 2004 to June 2018, demonstrated that the price-earnings index based on the IBOVESPA presents a non-linear trend and mean reversion, which contrasts with the EMH.

3 Data and empirical methodology

3.1 Data

The dataset analyzed in this study is the daily closing price (P_t) of the stock of ten companies listed on the São Paulo Stock Exchange Index (IBOVESPA), considering the theoretical asset portfolio in the first four months of 2020. The series are listed in Table 1. The study considers the period from January 1, 2016, to December 31, 2022. The data source is Investing.com (`www.investing.com`). The data are made available upon request. The daily returns r_t are obtained by calculating the first logarithmic difference in the stock market price P_t in day t, $r_t = \log(P_t) - \log(P_{t-1})$, for each time series.

This research works with two sets of companies: those with higher P/E ratios and those with lower P/E ratios, considering the P/E ratio of 2019 (the last business day of the year) before the COVID-19 pandemic. The corresponding P/E ratios are listed in Table 1. It is important to highlight that companies with a negative P/E ratio were removed from the sample, as were companies with higher or lower P/E ratios that did not have data for the entire analysis period.

Table 1: Variables, units, acronyms, P/E ratios, and sources

Variable	Unit	Acronym	P/E	Source
Higher P/E				
CVC Brasil Operadora e Agência de Viagens S.A.	Price	CVCB3	144.44	Investing.com
BRF Brasil Foods S.A.	Price	BRFS3	96.10	Investing.com
Cogna Educação S.A.	Price	COGN3	79.89	Investing.com
Ultrapar Participações S.A.	Price	UGPA3	75.91	Investing.com
Natura &Co Holding S.A.	Price	NTCO3	67.97	Investing.com
Usinas Siderúrgicas de Minas Gerais S.A.	Price	USIM5	55.88	Investing.com
Lower P/E				
Transmissora Aliança de Energia Elétrica S.A.	Price	TAEE11	10.72	Investing.com
Petróleo Brasileiro S.A.	Price	PETR4	10.40	Investing.com
Banco do Brasil S.A.	Price	BBAS3	9.23	Investing.com
Companhia Energética do Estado de Minas Gerais S.A.	Price	CMIG4	6.43	Investing.com
Centrais Elétricas Brasileiras S.A.	Price	ELET6	4.84	Investing.com
Equatorial Energia S.A.	Price	EQTL3	1.90	Investing.com

Notes: P/E ratio considers as a basis the companies listed on the IBOVESPA on the date in December 2019. Source: Own elaboration.

To Ray and Tsay (2000), the log transformation becomes a problem when zero or very small returns are encountered. To mitigate this problem, following Perron and Qu (2010), it is adopted $r_t + 0.00001$. The conclusions remain the same if the returns with absolute magnitudes below 0.00001 were eliminated. Besides, squared log returns are used as measure for volatility, as adopted by Taylor (1986), Crato and de Lima (1994), Stărică and Granger (2005), Hull and McGroarty (2014), among others.

Table 2 shows the basic descriptive statistics of daily returns (first difference of natural logarithms) over the entire sample period. For several returns, the distributions appear asymmetric because there are positive and negative skewness estimates. All return series have heavy tails and show a strong deviation from normality (the skewness and kurtosis coefficients are different from those of the standard normal distribution, which are 0 and 3, respectively). In addition, the Jarque-Bera (JB) test rejected the null hypothesis of normality at the 5% significance level.

Table 2 also presents the results of three unit root tests: Augmented Dickey-Fuller – ADF (Dickey and Fuller, 1981), Phillips-Perron – PP (Phillips and Perron, 1988) and Kwiatkowski-Phillips-Schmidt-Shin – KPSS (Kwiatkowski et al., 1992). The results reveal that all daily returns are stationary, that is, the hypothesis of the unit root is rejected at the 5% significance level. Stationarity in a time series does not exclude the possibility of serial correlation (Henry, 2002).

Table 2: Descriptive statistics, Jarque-Bera (JB) test and unit root tests

	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5
Mean	0.0004	−0.0001	0.0002	0.0006	0.0010	0.0019
Median	0.0003	−0.0003	−0.0016	0.0010	0.0003	0.0000
Std. dev.	0.0393	0.0281	0.0336	0.0271	0.0309	0.0381
Min.	−0.4264	−0.2190	−0.2341	−0.2393	−0.2766	−0.2378
Max.	0.2816	0.1518	0.1790	0.2111	0.1595	0.3075
Skewness	−1.3105	−0.4880	−0.4606	−0.5486	−0.5794	0.3403
Kurtosis	22.707	11.099	9.646	15.439	12.135	8.873
JB	28 614	4820	3261	11 291	6141	2533
ADF	−17.835	−26.744	−28.964	−15.589	−29.607	−28.611
PP	−40.871	−40.405	−42.990	−48.072	−43.395	−40.428
KPSS	0.5152	0.0896	0.2137	0.0653	0.4084	0.3003
N. obs.	1733	1733	1733	1733	1733	1733

	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Mean	0.0016	0.0018	0.0017	0.0019	0.0019	0.0018
Median	0.0015	0.0023	0.0018	0.0010	0.0017	0.0018
Std. dev.	0.0160	0.0336	0.0275	0.0281	0.0313	0.0177
Min.	−0.1963	−0.3514	−0.2367	−0.2339	−0.2136	−0.1138
Max.	0.0845	0.2017	0.1592	0.1660	0.2792	0.0816
Skewness	−1.2860	−2.0422	−0.7200	−0.6065	0.1702	−0.2905
Kurtosis	18.634	23.364	13.997	12.376	11.627	7.165
JB	18 175	31 230	8908	6474	5399	1282
ADF	−30.348	−22.380	−29.591	−29.228	−29.334	−29.542
PP	−43.750	−42.403	−42.604	−42.368	−41.281	−45.223
KPSS	0.0901	0.0947	0.1614	0.0355	0.0973	0.1252
N. obs.	1733	1733	1733	1733	1733	1733

Notes: 1) The normality test is the Jarque-Bera test which has a χ^2 distribution with 2 degrees of freedom under the null hypothesis of normally distributed errors. The 5% critical value is equal to 5.99; 2) Critical values of the ADF, PP and KPSS tests, at the 5% level of significance, are equal to −1.95, −2.86 and 0.463, respectively. Source: Own elaboration.

3.2 Empirical methodology

3.2.1 Long memory

As described by Tsay (2010), the autocorrelation function (ACF) for a stationary time series decays exponentially to zero as the lag increases. When the time series presents a unit root (i.e., it is nonstationary), the sample ACF converges to one for all fixed lags as the sample size increases (Chan and Wei, 1988; Tiao and Tsay, 1983). However, for some time series, the ACF slowly decays to zero at a polynomial rate, as the lags increase. These processes exhibit long-memory behavior (long-range dependence).

Baillie (1996) presented a wider definition of long memory, as follows: given a discrete time series process (x_t), $t = 1, \dots, T$, with autocovariance function γ_h, at lag h, in the time domain, the process exhibits long-range dependence if

$$\gamma_h \approx \Xi(h)h^{2d-1}, \quad \text{as } h \to \infty \tag{1}$$

where $\approx$ denotes approximate equality for large h, $d \neq 0$ is the fractional long-memory parameter, $\Xi(h)$ is a slowly varying function at infinity. Consequently, the autocorrelation function exhibits slow and hyperbolic decay.

In addition, in the frequency domain, considering $f(\lambda)$ as the spectral density of the long-memory series (x_t), with $t = 1, \ldots, T$, at frequency λ, then,

$$f(\lambda) \sim c\,\lambda^{-2d}, \quad \text{as } \lambda \to 0, \tag{2}$$

where c is a constant. In other words, $f(\lambda)$ is unbounded when the frequency is near zero (Baillie, 1996; Charfeddine, 2016).

In this context, the following definitions regarding the behavior of process $\{x_t\}$ can be described as follows: $d = 0$, short memory (or white noise); ii) $0 < d < 0.5$, stationary with long memory; iii) $0.5 \leq d < 1$, the process is mean reverting, even though it is not covariance stationary; and iv) if $d \geq 1$, nonstationary and does not present mean reversion. When $d = 1$, there is a non-stationary process characterized by the presence of a unit root. The process exhibits anti-persistence behavior when $d \in (-0.5,\ 0)$.

In the related literature, there are several estimators of the fractionally integrated parameter (d) that can be classified as parametric or semi-parametric. One of the most popular semiparametric estimators is the GPH estimator (Geweke and Porter-Hudak, 1983). In this study, this estimator was used for all estimates of the fractionally integrated parameter (d). The ELW estimator (Shimotsu and Phillips, 2005) was also adopted to ensure the robustness of the results. In addition, because the estimated fractionally integrated parameter may vary over time, a rolling estimation was adopted to capture the time variation of $\hat{d}$. In this case, the estimated parameter $\hat{d}$ is calculated for an initial time window (250-day window), and then the sample is rolled forward one point by eliminating the first observation, adding the next one, and then recalculating the parameter $\hat{d}$. The rolling window technique has some shortcomings, but it provides a good first proxy for the time variation of $\hat{d}$.

3.2.2 The GPH Estimator

The GPH employs the periodogram function $I(\lambda_j)$ as an estimate of the spectral density function ($f(\lambda)$) in Equation (2) at the frequency

$$\lambda_j = \frac{2\pi j}{T}, \quad j = 0, 1, \ldots, \left\lfloor \frac{T}{2} \right\rfloor$$

where T is the sample size and $\lfloor \bullet \rfloor$ denotes the integer part function. In this case, considering frequencies close to zero, Geweke and Porter-Hudak (1983) suggested the following approximation to estimate d – see Molinares et al. (2009), Charfeddine and Guégan (2012), and Charfeddine (2016):

$$\ln\{I(\lambda_j)\} = \beta - d\ln\left\{4\sin^2\left(\frac{\lambda_j}{2}\right)\right\} + \varepsilon_j, \quad j = 1, 2, \ldots, g, \tag{3}$$

where $\{\varepsilon_j\}$ is the white noise and g is the bandwidth ($g = T^\alpha$, with T equal to the number of observations, and $0 < \alpha < 1$), which corresponds to the number of frequencies used in the regression of Equation (3). Under certain conditions, the GPH estimator is consistent and asymptotically normally distributed. For asymptotic properties of the estimator, see Hurvich et al. (1998) and Velasco (2000).

3.2.3 The Exact Local Whittle (ELW) estimator

Shimotsu and Phillips (2005) developed a semiparametric estimator, ELW (here called $\hat{d}_{ELW}$), to estimate the fractionally integrated parameter (d). This estimator ($\hat{d}_{ELW}$) is consistent and has the same normal limit distribution for all values of d if the optimization covers an interval of width less than 9/2 and the mean (initial value) of the process is known (for details, see Shimotsu and Phillips, 2005; Shimotsu, 2010). This approach allows a substantial range of stationary and non-stationary possibilities for d. Therefore, ELW is a better estimator than GPH, since GPH is not good general-purpose estimator when the value of d extends into the nonstationary zone beyond 3/4 (Kim and Phillips, 2006; Shimotsu and Phillips, 2005).

The idea is to estimate (G, d) by minimizing the objective function

$$Q_g(G,d) = \frac{1}{g}\sum_{j=1}^{g}\left[\ln\left(G\lambda_j^{-2d}\right) + \frac{1}{G}I_{\Delta^d x_t}(\lambda_j)\right] \tag{4}$$

where $I_{\Delta^d x}(\lambda_j) = \frac{1}{2\pi T}\left|\sum_{t=1}^{T}\Delta^d x_t\,\exp(i\lambda_j t)\right|^2$ is the periodogram of $\Delta^d x_t$, $\Delta^d = (1-B)^d$ and B is the lag operator.

By concentrating $Q_m(G, d)$ with respect to G, the estimated value for d ($\hat{d}_{ELW}$) obtained using this method is:

$$\hat{d}_{\text{ELW}} = \underset{d\in[d_1,d_2]}{\arg\min}\, R(d) \tag{5}$$

where d_1 and d_2 are the lower and upper bounds of the admissible values of d so that $-\infty < d_1 < d_2 < \infty$. Furthermore,

$$R(d) = \ln\hat{G}(d) - 2d\frac{1}{g}\sum_{j=1}^{g}\ln\lambda_j \quad \text{and} \quad \hat{G}(d) = \frac{1}{g}\sum_{j=1}^{g}I_{\Delta^d x_t}(\lambda_j). \tag{6}$$

where g is a truncation parameter. The ELW estimator is a consistent, asymptotically normally distributed alternative.

4 Results and discussions

As previously described, most estimates of the fractional parameter d were performed using the GPH estimator. An ELW[1] estimator was also used to ensure robustness. Furthermore, because the estimated fractionally integrated parameter may vary over time, a rolling estimation was adopted. As it is impractical to report all the estimated parameters ($\hat{d}$) for each rolling window, the results are presented using graphical depictions and tables with general descriptive statistics. In all methods, this study used $g = T^{0.7}$ as the bandwidth (details of the choice of bandwidth can be consulted at Reisen, 1994; Lee and Robinson, 1996; Hurvich et al., 1998 and Diebold and Inoue, 2001). Other bandwidths, such as $g = T^{0.5}$, $g = T^{0.6}$ and $g = T^{0.8}$ generated similar results.

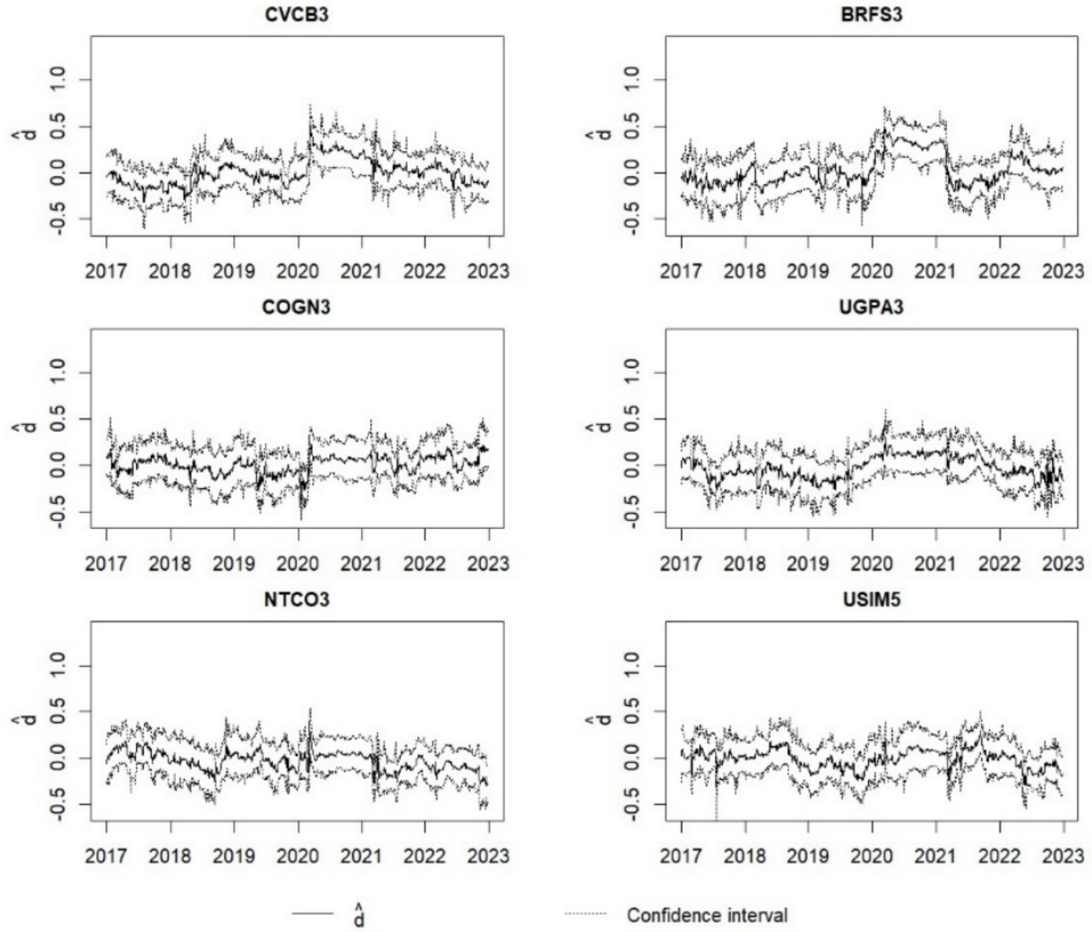
First, Figures 1 and 2 present the estimated time-varying fractional parameter ($\hat{d}$) for the returns of companies with higher P/E and lower P/E, respectively. In this step, the GPH estimator was adopted. Looking more specifically at the P/E ratio, the results are similar for both groups of companies, that is, a lower ratio does not appear to guarantee that returns will be persistent. Thus, information about the P/E ratio appears to be reflected quickly in the market, as postulated by the semi-strong form of the EMH. However, considering the period of COVID-19, at least for the returns of some companies, the findings of this research are also partially like those found by some international studies related to the pandemic. In other words, in some cases, there was an increase in persistence (increase in the value of the fractional parameter), but the long memory disappeared, which supports AMH.

Now, looking at the results more broadly, it is possible to note that the estimated parameter $\hat{d}$ was time varying in nature, which is unsurprising. However, in general, for both groups of companies, the estimated value of d is close to zero, and the long-range dependence hypothesis can be rejected, that is, the estimates do not find long-memory behavior over a major part of the sample. A few exceptions occurred during the most turbulent period of the COVID-19 pandemic, in which the possibility of long-range dependency on the returns of some companies (for instance, CVCB3 and BRFS3, from the group with higher P/E) was not rejected. Nevertheless, even in these cases, the estimated parameter is in the range of 0 to 0.5 ($0 < \hat{d} < 0.5$), thereby indicating mean reversion and transitory effects.

To complement the analysis, Table 3 shows the descriptive statistics of the estimated time-varying long-memory parameter ($\hat{d}$), considering the entire period. The ELW estimator was also presented. Notably, in this case, the GPH and ELW estimators presented

[1] The Two-Step Exact Local Whittle – 2SELW (Shimotsu, 2010) estimator was also used. The results were similar to those of the GPH and ELW estimators and are available on request.

Figure 1: Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (based on GPH) for the returns of companies with higher P/E

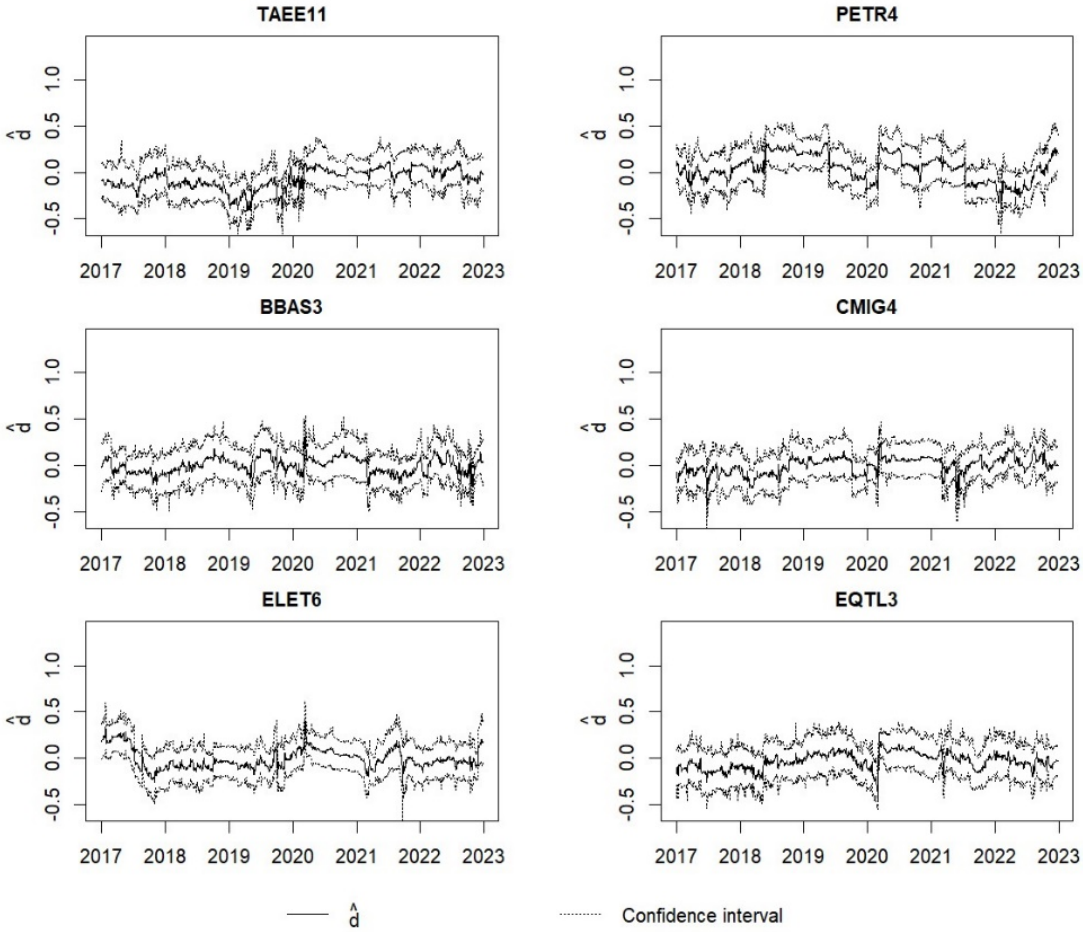


Notes: 1) In the estimates of d, $g = T^{0.7}$ is considered, where T is the number of observations; 2) Dashed lines correspond to the 95% confidence interval. Source: Own elaboration.

similar results. It is observed that although some maximum values are close to 0.5, which occurred during periods of turbulence, such as election periods and the COVID-19 pandemic, the medians are very close to zero (medians are more interesting than means because the estimated parameter $\hat{d}$ is not normally distributed). Furthermore, even when observing the values of the third quartile, the estimated values were still very close to zero.

In short, the P/E ratio does not appear to be a factor that enables the persistence of returns. Additionally, in general, except for the period of the COVID-19 pandemic, the results align with those found in the literature on returns; see, for example, Cavalcante and Assaf (2004), Hull and McGroarty (2014) and Bhattacharya et al. (2018). These authors did not find (or find little evidence) of long memory in return series for emerging markets, including Brazil. Other works, which examined several international stock markets, also did not find evidence (or strong evidence) of long-range dependence, such as: Lo (1991),

Figure 2: Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (based on GPH) for the returns of companies with lower P/E



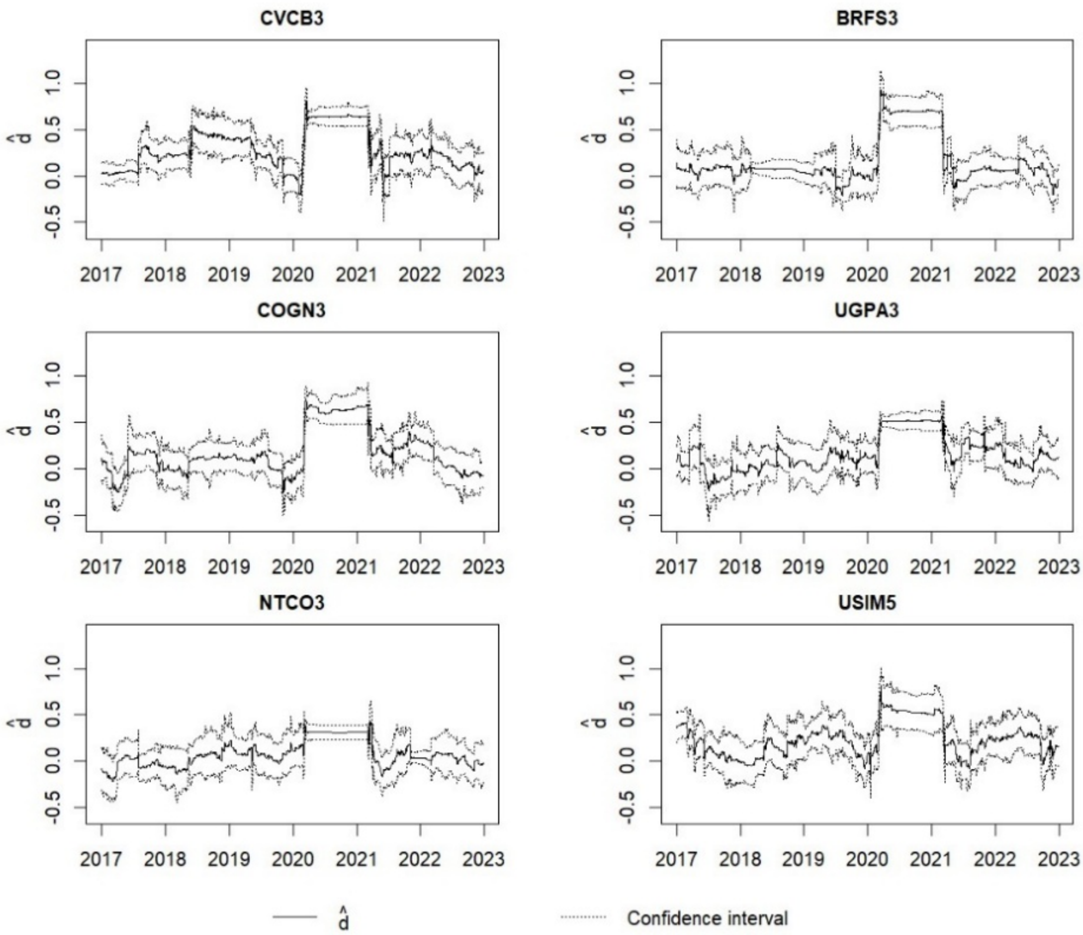
Notes: 1) In the estimates of d, $g = T^{0.7}$ is considered, where T is the number of observations; 2) Dashed lines correspond to the 95% confidence interval. Source: Own elaboration.

Jacobsen (1995), Crato and Ray (2000), Serletis and Rosenberg (2007), and Lu and Perron (2010).

In Figures 3 and 4 the results for volatility are presented. It is possible to see the time-varying fractional coefficient ($\hat{d}$), for companies with higher P/E and lower P/E, respectively (the GPH estimator is used). More specifically, regarding the P/E ratio, the results for volatility follow the findings for returns, that is, P/E ratio alone does not appear to lead to volatility persistence. Nonetheless, during the COVID-19 pandemic, for the volatility of all companies, there was a strong increase in persistence (increase in the value of the fractional parameter), again with mean reversion. Therefore, for the sample companies used in this research, the characteristics of both EMH and AMH can be observed.

Furthermore, looking at the results more generally, as in the case of returns, the fractional parameter also varies with time. However, there is a substantial difference in volatility. As can be seen, in many cases, the estimated parameter is far from zero and it is statistically significant, that is, the long-range dependence hypothesis cannot be rejected. Thus, in some periods, there is long-memory behavior. Specifically, these intervals correspond to times of turmoil, either from domestic sources (e.g., presidential elections, scandals of corruption, or political interference in companies linked to the government) or from foreign sources, such as the recent crisis generated by the COVID-19 pandemic. It is noteworthy that during the COVID-19 pandemic the estimated parameter $\hat{d}$ reached significant values, between 0.5 and 1 ($0.5 \leq \hat{d} < 1$), revealing periods in which volatility did not show stationary covariance, yet with mean reversion. In addition, the long-range dependence was transitory and disappeared.

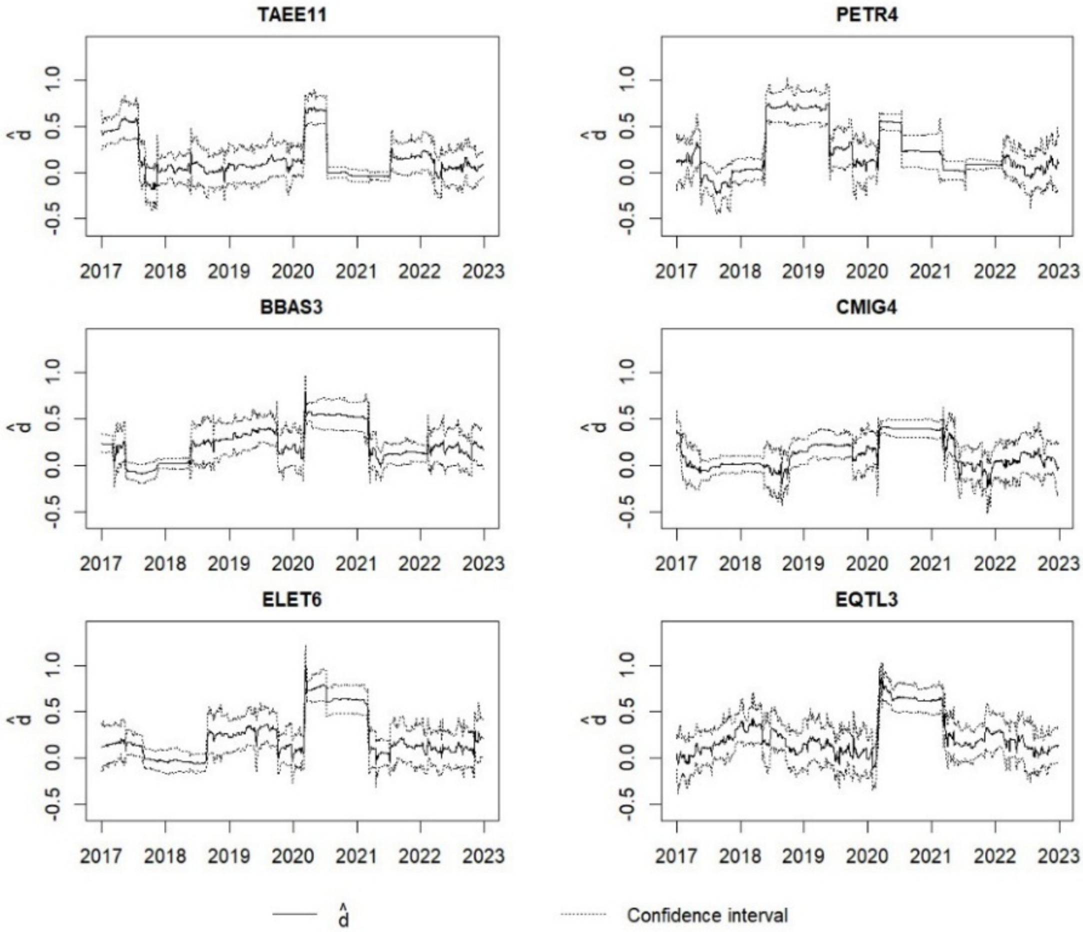
Figure 3: Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (based on GPH) for the volatilities of companies with higher P/E



Notes: 1) In the estimates of d, $g = T^{0.7}$ is considered, where T is the number of observations; 2) Dashed lines correspond to the 95% confidence interval. Source: Own elaboration.

In Table 4, the descriptive statistics of the estimated time-varying long-memory parameter ($\hat{d}$) for volatilities are demonstrated, considering the entire period. The ELW estimator was also adopted. One can observe much higher median values than for the

Figure 4: Time-varying fractional parameter ($\hat{d}$) using rolling estimation and confidence interval (based on GPH) for the volatilities of companies with lower P/E



Notes: 1) In the estimates of d, $g = T^{0.7}$ is considered, where T is the number of observations; 2) Dashed lines correspond to the 95% confidence interval. Source: Own elaboration.

returns, and, in relation to the maximum value of the fractional parameter, for almost all companies, it presented a result higher than 0.5 ($0.5 \leq \hat{d} < 1$), again, largely due to the COVID-19 pandemic. For some estimators, the values were greater than unity ($\hat{d} > 1$). The third quartile also showed significant values, which were higher than those observed for returns.

It is important to note that while for returns, only for a few companies and periods there was long-range dependence during the COVID-19 pandemic, in the case of volatility, the COVID-19 pandemic heightened the level of persistence. The findings of this research are in line with some works that found evidence of persistence in volatility in some financial markets, especially in periods of turmoil. See Ding et al. (1993), Bollerslev and Mikkelsen (1996), Baillie et al. (1996), Breidt et al. (1998), Crato and Ray (2000), Caporale et al. (2019), among others. Monte (2023) showed strong evidence of long-memory dependence in the CBOE Brazil ETF volatility index, especially during the COVID-19 pandemic.

5 Conclusions

The focus of this research is to verify whether there is short- or long-memory behavior in returns and volatilities and whether the behavior is similar for the two groups of companies listed on IBOVESPA: with higher and with lower price-earnings (P/E) ratios. In general, for returns and volatility, the results are very similar for both groups of companies, that is, the lower P/E ratio alone does not appear to lead to persistent heterogeneity of returns and volatility.

The long-memory behavior when it occurs, as in the COVID-19 pandemic, especially for volatility, is not constant over time; it is transitory and eventually disappears. For volatility, there seems to be a cyclical pattern of market efficiency/inefficiency, with inefficiency arising from periods of negative turbulence (domestic or external to the Brazilian economy). However, in all cases, there is a convergence toward efficiency, which can be explained, for example, by the increase in computational capacity and the speed at which information is disseminated to agents (dos Santos et al., 2024).

Therefore, some main considerations can be derived from the results, and it may be said that they are in line with the authors who claim that the AMH can reconcile the EMH with all its behavioral alternatives. One of the practical implications of AMH is that profit opportunities arise occasionally depending on the degree of market efficiency and market conditions. This can be seen during some periods, especially for volatility and the COVID-19 pandemic. This suggests that the pandemic pushed the returns of some stocks into inefficiency.

In addition, as described by Malkiel (2003), some market participants are demonstrably fewer than rational. Thus, pricing irregularities and even predictable patterns in stock returns can appear over time and persist for short periods. However, according to the author, the result will not be the abandonment of the belief of many in the profession that the stock market is remarkably efficient in using information. Furthermore, Malkiel (2003) states that efficient financial markets do not allow investors to earn above-average returns without accepting above-average risk. In this study, especially for volatility, there is great persistence in moments of negative turbulence, but this persistence becomes transitory.

It is therefore essential to consider that, although market participants, such as institutional investors, investment funds, pension funds, and individual investors, may employ strategies and instruments to benefit from periods of heightened persistence in return volatility, the potential gains from such operations are non-recurring. One example of such instruments includes futures and options contracts (derivatives) (Hull, 2021).

The purchase of *put options* can serve as *hedging instruments*: puts deliver significant returns when prices and returns (of most investors) fall substantially (usually accompanied by increased volatility). This was precisely the initial scenario triggered by the COVID-19 shock: a sharp decline in asset prices and median investor's return. However, those who were positioned in put options or engaged in short-selling strategies achieved considerable profits, thus at least mitigating general losses (which is one of the main functions of hedging strategies, that is, work as a type of insurance). Still, it is important to note that all of these strategies entail costs, meaning that systematically holding them can reduce long-term portfolio returns and may not be suitable for all investor profiles or objectives.

A possible extension of this study is to analyze market efficiency for other groups of companies, considering specific industrial sectors, for example, or even using other metrics to separate groups, such as the size of dividend yields and market capitalization. Another extension of this work could be to investigate market efficiency for different frequencies (e.g., weekly and monthly). In addition, checking for possible structural changes (structural breaks) in the time-varying estimated fractional parameter ($\hat{d}$) may be a future success.

Use of AI tools declaration

The authors declare they have not used Artificial Intelligence (AI) tools in the creation of this article.

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Conflict of interest

All authors declare no conflicts of interest in this paper.

Data availability

Data will be made available on request.

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Table 3: Descriptive statistics of the estimated time-varying long memory parameter ($\hat{d}$) for returns

GPH

	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5
Median	−0.0112	−0.0151	0.0141	−0.0410	−0.0198	−0.0004
Mean	0.0012	0.0252	0.0057	−0.0251	−0.0344	−0.0115
Min.	−0.3206	−0.2633	−0.3610	−0.2885	−0.3028	−0.2940
Max.	0.5581	0.4850	0.2373	0.3009	0.2821	0.2916
1st Quart.	−0.0987	−0.0931	−0.0530	−0.1100	−0.1106	−0.0803
3rd Quart.	0.0667	0.0825	0.0711	0.0840	0.0342	0.0545
Variance	0.0182	0.0266	0.0077	0.0131	0.0094	0.0096
St. Dev.	0.1350	0.1630	0.0879	0.1144	0.0971	0.0982
Skewness	0.5931	0.9059	−0.3850	0.0546	−0.1389	−0.2700
Kurtosis	3.1111	2.7781	3.0193	1.9947	2.6743	2.7532
JB	88.062	206.45	36.798	62.933	11.204	21.740

	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	−0.0594	0.0554	−0.0056	0.0065	−0.0333	−0.0294
Mean	−0.0684	0.0476	−0.0038	−0.0052	−0.0101	−0.0326
Min.	−0.4311	−0.3910	−0.2349	−0.3849	−0.2985	−0.3644
Max.	0.1270	0.3241	0.3648	0.2566	0.4271	0.1813
1st Quart.	−0.1376	−0.0713	−0.0717	−0.0715	−0.0860	−0.0961
3rd Quart.	0.0179	0.1693	0.0562	0.0625	0.0452	0.0324
Variance	0.0120	0.0207	0.0074	0.0080	0.0115	0.0076
St. Dev.	0.1095	0.1438	0.0862	0.0896	0.1072	0.0873
Skewness	−0.6767	−0.1073	0.1771	−0.4323	0.8214	−0.2106
Kurtosis	3.1751	2.1605	2.9688	3.1488	3.4486	2.7107
JB	115.55	46.155	7.821	47.791	180.03	16.047

ELW

	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5
Median	−0.0008	0.0038	−0.0160	−0.0241	0.0013	0.0039
Mean	−0.0021	0.0436	−0.0180	−0.0078	−0.0087	−0.0067
Min.	−0.2665	−0.1619	−0.1935	−0.1692	−0.2278	−0.2106
Max.	0.5383	0.4291	0.2277	0.1831	0.2373	0.2285
1st Quart.	−0.0803	−0.0677	−0.0786	−0.0617	−0.0723	−0.0715
3rd Quart.	0.0384	0.0957	0.0509	0.0520	0.0574	0.0525
Variance	0.0152	0.0206	0.0056	0.0049	0.0074	0.0069
St. Dev.	0.1232	0.1435	0.0752	0.0700	0.0862	0.0830
Skewness	0.4931	1.0407	0.0175	0.4175	−0.1101	−0.0858
Kurtosis	3.0583	2.9609	2.2382	2.0291	2.4708	2.4992
JB	60.550	268.69	35.697	101.22	20.117	17.134

	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	−0.0656	0.0320	0.0242	0.0389	−0.0222	−0.0464
Mean	−0.0642	0.0561	0.0140	0.0160	−0.0104	−0.0278
Min.	−0.2446	−0.3106	−0.2164	−0.1751	−0.2053	−0.3066
Max.	0.1395	0.4296	0.3797	0.2479	0.4332	0.1866
1st Quart.	−0.1543	−0.0395	−0.0167	−0.0466	−0.0680	−0.0850
3rd Quart.	0.0320	0.1280	0.0477	0.0786	0.0391	0.0425
Variance	0.0093	0.0147	0.0032	0.0052	0.0069	0.0054
St. Dev.	0.0962	0.1212	0.0569	0.0719	0.0830	0.0736
Skewness	−0.0358	0.2491	−0.0880	−0.4844	0.8801	0.1104
Kurtosis	1.4226	2.1671	6.2550	2.2137	4.1146	2.3483
JB	153.90	58.015	660.89	96.165	269.72	29.051

Source: Own elaboration.

Table 4: Descriptive statistics of the estimated time-varying long memory parameter ($\hat{d}$) for volatilities

GPH

	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5
Median	0.2330	0.0790	0.1276	0.1035	0.0515	0.2223
Mean	0.2828	0.1589	0.1757	0.1614	0.0710	0.2326
Minimum	−0.2486	−0.2187	−0.3220	−0.2170	−0.2124	−0.1587
Maximum	0.8174	0.9348	0.7517	0.5779	0.4065	0.8229
1st Quart.	0.1281	0.0332	−0.0055	0.0408	−0.0209	0.0895
3rd Quart.	0.4127	0.1057	0.2511	0.2382	0.1253	0.3211
Variance	0.0438	0.0652	0.0582	0.0351	0.0178	0.0321
St. Dev.	0.2092	0.2554	0.2412	0.1874	0.1335	0.1792
Skewness	0.4395	1.5297	0.8895	0.7500	0.5775	0.4682
Kurtosis	2.3280	3.9126	2.8808	2.6604	2.6024	2.4473
JB	75.571	632.41	197.05	146.47	92.306	73.035

	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	0.0785	0.1245	0.1976	0.0879	0.1380	0.1715
Mean	0.1387	0.2229	0.2248	0.1233	0.2119	0.2343
Minimum	−0.2000	−0.2360	−0.0920	−0.2662	−0.1217	−0.1782
Maximum	0.7552	0.7534	0.7901	0.4258	0.9708	0.8656
1st Quart.	0.0032	0.0388	0.1161	0.0098	0.0630	0.0974
3rd Quart.	0.1571	0.2914	0.3375	0.2262	0.2805	0.3088
Variance	0.0411	0.0672	0.0332	0.0253	0.0523	0.0427
St. Dev.	0.2028	0.2591	0.1822	0.1590	0.2287	0.2067
Skewness	1.5611	0.8706	0.3930	0.4630	1.1330	1.0350
Kurtosis	4.4651	2.4987	2.4144	2.2029	3.3285	3.1441
JB	738.29	203.32	59.280	92.17	325.24	267.03

ELW

	CVCB3	BRFS3	COGN3	UGPA3	NTCO3	USIM5
Median	0.2268	0.0597	0.1173	0.1294	0.0804	0.1806
Mean	0.2802	0.1472	0.1779	0.1856	0.1012	0.2142
Minimum	−0.2497	−0.3978	−0.3965	−0.3304	−0.4071	−0.2965
Maximum	0.8628	0.9087	0.9489	1.6732	1.0362	0.7490
1st Quart.	0.1444	0.0237	0.0504	0.0839	0.0201	0.1006
3rd Quart.	0.3692	0.0983	0.1757	0.2194	0.1131	0.2879
Variance	0.0352	0.0602	0.0395	0.0295	0.0131	0.0188
St. Dev.	0.1876	0.2454	0.1988	0.1717	0.1144	0.1371
Skewness	0.8162	1.5563	1.3798	1.4537	1.4257	0.7597
Kurtosis	2.6170	3.9812	3.7167	7.3605	7.1250	2.8661
JB	174.11	660.86	504.36	1705.9	1561.8	144.17

	TAEE11	PETR4	BBAS3	CMIG4	ELET6	EQTL3
Median	0.0747	0.1102	0.1932	0.0919	0.1366	0.1516
Mean	0.1359	0.2433	0.2273	0.1376	0.1976	0.2065
Minimum	−0.3436	−0.4227	−0.3618	−0.3778	−0.3061	−0.2286
Maximum	0.7517	1.0896	1.8146	0.7876	1.0995	1.0170
1st Quart.	0.0418	0.0661	0.1250	0.0293	0.0579	0.0742
3rd Quart.	0.1434	0.2807	0.3058	0.2103	0.2352	0.2244
Variance	0.0274	0.0605	0.0266	0.0186	0.0428	0.0342
St. Dev.	0.1655	0.2459	0.1631	0.1365	0.2068	0.1850
Skewness	1.6685	1.0343	1.5469	0.7303	1.3216	1.3127
Kurtosis	4.7728	2.6615	10.7677	3.0149	3.8799	3.7335
JB	886.29	272.22	4340.7	132.28	481.67	461.09

Source: Own elaboration.