

---

# Modelling Nonlinear Dynamics of Oil Futures Market

Ayben Koy \*♣

♣Istanbul Ticaret University, Department of Banking and Finance

Submitted: June 2, 2016 • Accepted: February 20, 2017

---

**ABSTRACT:** Due to the fact that oil prices had a falling outlook after the global crisis, modeling oil market prices has been a topic of interest among researchers. The goals of this study are to investigate the recession or growth periods of oil futures markets using Markov switching autoregressive models, and to analyze the models' durations and probabilities to provide information to the investors who invest in these markets. The study findings indicate that oil prices have a nonlinear pattern with three regimes. The model that best describes the oil futures markets is MSIH(3)-AR(0) with three regimes.

---

**JEL classification:** G10, G15

**Keywords:** oil futures, Markov switching, regime switching, regime dependence

## Introduction

The study of oil prices has been growing in importance in recent years because of the falling outlook. The extreme movements of both oil and oil futures prices as well as oil prices beginning in 2008 sparked the interest. 2009 was keenly watched after the oil prices exceeded \$145 and then sharply decreased below \$50 in 2008. The oil futures prices never reached \$100 USD after July 2014, and the following year prices fell. According to the Organization of the

---

\*Corresponding Author. Email: Akoy@ticaret.edu.tr

Petroleum Exporting Countries' (OPEC) Annual Report 2015, crude prices fell by around 50%, The OPEC Reference Basket averaged just under \$50 per barrel in 2015, and as a result many investments were deferred and some were cancelled, with exploration and production spending falling by around 20% compared to 2014.

This research aims to analyze oil futures prices following the global crisis by using Markov switching autoregressive models (MS-AR) and to bring to light the oil futures market. The models examine the futures prices from a nonlinear perspective. Additionally, the information about the duration and probabilities of the regimes in the nonlinear switching models is important for investors in these markets. Taking a look at the literature, it can be seen that many studies related to the price dynamics of futures markets exist. What makes this study different from those others is that it examines the prices using Markov regime switching models which also provide detailed information about market performance for international investors.

## 1 Literature

The theoretical basis for the observed price behavior of futures was examined by Working (1949), with a particular focus on *inter-temporal price relations*, “defined as relations *at a given time* between prices applicable to different times” (Working, 1949, p.1254). Working found that the inter-temporal price relation was explained by the commodity’s carrying cost because participants in the futures market try to make profit, the arbitrage possibilities eliminate any bias in the futures prices. In another early study, Rockwell et al. (1967), examined 25 commodity markets for the period 1947–1965 and found that futures prices rose by about 4% annually. Breeden and Litzenberger (1978), and Breeden (1979, 1980) modeled expected returns in commodity futures by using *intertemporal capital arbitrage pricing model* (CAPM). They found that changes in the expected returns were associated with real consumption and the consumption betas. In addition, Dusak (1973) and Breeden (1980) also related futures prices to the expected risk premium.

One recent study examines price discovery in the prominent market price benchmarks crude oil West Texas Intermediate (WTI) and crude oil Brent (Elder et al., 2014). The authors find no evidence that the dominant role of crude oil WTI in price discovery was diminished by the price spread between WTI crude oil and Brent crude oil that emerged in 2008. In another study, the volatility of crude oil price futures returns were analyzed by Baum and Zerilli (2016) for the period from October 2001 to December 2012. The results of the model applied to the intraday data indicate that stochastic volatility models are effective in fitting the volatility of oil price futures returns. Bernard et al. (2015) modeled oil futures prices using weekly and monthly data and maturities of one to four months. Their

results show that forecast performances improve with longer date-to-maturity futures, suggesting that the role of the convenience yield is greater when physical oil inventories are held for longer durations. In addition, forecast accuracy is highest at the one year horizon, though the time-varying convenience models (the mean-reverting class of models from Schwartz and Smith (2000) and Schwartz (1997)) have a much higher accuracy than the autoregressive conditional heteroskedasticity (ARCH) and generalized autoregressive conditional heteroskedasticity (GARCH) models over the three and five-year horizons.

Even though there is not a wide literature on the nonlinear approach, there have been some studies examining spot oil prices by Vo (2009), Kordnoori et al. (2013), and Zlatcu et al. (2015). Vo (2009) used the Markov switching stochastic volatility (MSSV) model to explain the behavior of crude oil prices in order to forecast their volatility. Zlatcu et al. (2015) studied the fuel markets of Romanian, Germany, France, Poland and the Czech Republic, examining the price volatility and the response of retail fuel prices to changes in international crude oil prices by estimating univariate and multivariate GARCH models, as well as applying a momentum threshold autoregressive (MTAR) co-integration model. Kordnoori et al. (2013) modelled the fluctuations of Brent oil prices by integrating the limit probability distribution of a Markov chain and Gumbel Max distribution.

One of the previous studies examining the nonlinear behavior of oil futures considers the daily returns of the second nearest crude oil futures based on WTI: Fong and See (2002) examine the effect of volatility in daily returns on crude oil futures using GARCH, RS, RSARCH-t and RSGARCH-t models. They find that RS models are useful both to financial historians interested in studying the factors behind the evolution of volatility and to oil futures traders interested in using the model to extract short-term forecasts of conditional volatility. Similarly, Chevallier (2013) used the Markov regime switching vector autoregression (MS-VAR) model to explore the influence of different economic variables on the 2008 oil price swing. Zhang and Zhang (2015) singled out a Markov switching model (MS(3)-AR(2)) with three regimes for the samples of Brent and WTI crude oil prices before and after the 2008 financial crisis. They found three regimes for both crude oil markets before and after the crisis.

## 2 Data and Model

### 2.1 Data

The data used includes daily crude oil WTI futures (1539 days) and crude oil Brent futures (1592 days) from January 4, 2010 to December 31, 2015 (Figure 1). For the futures oil prices, we used the nearest contracts' prices of WTI. The data is obtained from investing.com, which

is a global financial portal that offers 27 localized editions in 20 languages. We used the differential natural logarithmic prices (logarithmic returns).

One crude oil WTI futures or crude oil Brent futures contract is equivalent to 1000 US barrels (42 000 gallons) of light, sweet crude oil. These contracts are some of the world's most liquid oil commodities and are important benchmark for oil markets.

Table 1 shows that the skewness is negative for both WTI and Brent. This negative skew indicates that the tail on the left side of the probability density function is fatter than that on the right side, which has already been seen in Figure 1. Similarly, the kurtosis is less than 3 for both series – these series are said to be *platykurtic*. These distributions produce fewer and less extreme outliers than do normal distributions.

## 2.2 Markov Switching Autoregressive Model

The Markov Regime Switching model is used to describe a situation or stochastic process that determines the change from one regime to another via a Markov chain. In Markov switching models, the Markov chain is used to model the behavior of a state variable that cannot be directly observed and determines the regime of the market. The regime of the state variable should be strongly related to the regime of the economy or market. The economy might be in a recession regime or a growth regime. In a Markov Regime Switching model, the state (regime) of the economy ( $s_t$ ) cannot be directly observed, although the time series variable  $y_t$  can be observed. The state of the market can only be identified with some probability. At the same time those observations are supposed to be dependent on the properties of the regime. When the state of the economy in the Markov regime is determined, the next regime can be expressed as a probability.

The general idea behind this class of RS models is that there is a time series process  $y_t$  dependent on an unobservable regime variable  $s_t$  that represents the probability of being in a particular state of the world (Krolzig, 2000). This can be written as follows:

$$p(y_t|Y_{t-1}; X_t; s_t) = \begin{cases} f(y_t|Y_{t-1}; X_t; \theta_1) & \text{if } s_t = 1 \\ \vdots & \\ f(y_t|Y_{t-1}; X_t; \theta_m) & \text{if } s_t = m \end{cases} \quad (1)$$

where  $X_t$  are exogenous variables; and  $\theta_m$  is the parameter vector associated with regime  $m$ .

In Markov switching models, the regime-generating process is an ergodic Markov chain

with a finite number of states defined by the transition probabilities (Krolzig, 2000).

$$p_{i,j} = Pr(s_{t+1} = j | s_t = i); \quad \sum_{j=1}^m p_{i,j} = 1 \text{ and } i, j = \{1, \dots, m\} \quad (2)$$

$s_t$  follows an ergodic M-state Markov process with an irreducible transition matrix:

$$\mathbf{P} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,m} \\ \vdots & \ddots & \vdots \\ p_{m,1} & \cdots & p_{m,m} \end{bmatrix} \quad (3)$$

In a two-state model, the transition probabilities of moving from one state to the other are denoted as:

$$\begin{aligned} Pr(s_{t+1} = 1 | s_t = 1) &= p_{1,1} \\ Pr(s_{t+1} = 2 | s_t = 1) &= p_{1,2} \\ Pr(s_{t+1} = 1 | s_t = 2) &= p_{2,1} \\ Pr(s_{t+1} = 2 | s_t = 2) &= p_{2,2} \end{aligned} \quad (4)$$

Note that for each  $p_{i,j}$  to define a proper probability, it must be nonnegative, and it should also hold that  $p_{11} + p_{12} = 1$  and  $p_{21} + p_{22} = 1$  (Franses and van Dijk, 2000).

The probability of which regime is in operation at time  $t$  conditional on the information at time  $t - 1$  only depends only on the statistical inference on  $s_{t-1}$ :

$$Pr(s_t | Y_{t-1}; X_t; S_{t-1}) = Pr(s_t | s_{t-1}) \quad (5)$$

If the probabilities of switching between regimes are known, the ergodic probability of any state can be calculated. The probability of any observation being in any state is called the ergodic probability. The ergodic probabilities for a two-state model are given as (Bildirici et al., 2010):

$$Pr(s_t = 1) = \frac{1 - p_{2,2}}{1 - p_{1,1} - p_{2,2}}, \quad Pr(s_t = 2) = \frac{1 - p_{1,1}}{1 - p_{1,1} - p_{2,2}} \quad (6)$$

The Markov switching time series analysis was first implemented by Hamilton (1989) to analyze business cycles. Hamilton investigated the possibility that macroeconomic variables fluctuate on a cyclical time scale between calendar time (e.g., month, quarter or another unit of time) and economic time. The time transformations between economic and calendar time depend on the economic history of the process such as whether the economy has been

in a cyclical recession or growth phase. The two main types of Markov switching models are the Markov switching model of conditional mean (MSM) and the Markov switching intercept (MSI) model. In the MSM model, the regime switches according to the conditional mean ( $\mu_{s_t}$ ), while in the MSI model, the regime switches according to the constant ( $c_{s_t}$ ). In addition, the Markov switching intercept and heteroskedasticity (MSIH) Model is a third type of Markov switching model that has proven to be strong in explaining financial time series. These three models are denoted as follows:

$$\text{MSM model :} \quad y_t - \mu_{s_t} = \Phi(y_{t-1} - \mu_{s_{t-1}}) + u_t \quad (7)$$

$$\text{MSI model :} \quad y_t - c_{s_t} = \Phi \cdot y_{t-1} + u_t \quad (8)$$

$$\text{MSIH model :} \quad y_t - c_{s_t} = \Phi \cdot y_{t-1} + u_t \cdot \sigma_{s_t} \quad (9)$$

where  $\Phi$  is an autoregressive coefficient;  $u_t$  is an unobservable zero-mean white noise vector process;  $y_{t-1}$  is the lagged value of the dependent variable; and  $\sigma_{s_t}$  is the standard deviation of the error term conditional on the state variable  $s_t$ .

The regime in MS-AR models is defined as an unobserved state variable affecting the levels or volatility of the distributions of financial asset returns (Perez-Quiros and Timmermann, 2001; Guidolin and Timmermann, 2006). These models also explain the fat tails, periods of turbulence followed by periods of low volatility, and skewness of many financial series. Moreover, these models can capture nonlinear stylized dynamics of asset returns in a framework based on linear specifications, or conditionally normal or log-normal distributions, within a regime (Ang and Timmermann, 2011).

### 3 Empirical Analysis

We applied autoregressive models with different numbers of regimes (2 or 3) and different lags (0–4) to the time series of the oil futures logarithmic returns. Taking linearity as our null hypothesis, and following Davies (1987), we considered a p-value less than 0.05 a statistically significant rejection of the null hypothesis. We estimated 15 models for crude oil WTI and 14 models for crude oil Brent that show non-linear characteristics.

#### 3.1 Markov Switching Autoregressive Model for Crude Oil WTI

The model that best explains the nonlinearity of crude oil WTI is MSIH(3)-AR(0), which has the minimum Hannan-Quinn criterion (HQ) (−5.2666) and Schwarz criterion (SIC) (−5.2404) from among the models tested. This model has a large log-likelihood ratio statistic (4063.3734) that signifies the maximum likelihood estimate of the transition possibilities

for the Markov model. Additionally, the MSIH(3)-AR(0) model has the largest value of the likelihood ratio (LR) statistics (370.1262) which indicates how much more the nonlinear model explains the relation than does the linear model. Of the 15 models that shows statistically significant nonlinearity, the MSIH(3)-AR(0) is the most powerful one. The regime switching mechanism in the MSIH model is specified by the intercept (I) and volatility/heteroskedasticity (H).

Coefficients of the MSIH(3)-AR(0) model for crude oil WTI are shown in Table 3. The estimation procedure implemented in the “Ox Metrics program” identifies regime 1 (slightly downward), regime 2 (slightly upward) and regime 3 (sharply downward) of the model. In many studies, regime 3 is identified as an expansion period with positive coefficients appropriate to the business cycle (Krolzig, 2000; Markov, 2010; Medhioub, 2015; Koy, 2017). Due to the falling trend across the whole observation period, we have negative coefficients in regime 1 and regime 3. With negative returns and high volatility, regime 1 is called *slightly downward* by Zhang and Zhang (2015), while regime 3 with the highest volatility and absolutely negative returns, is called *sharply downward*. Regime 2, which has the only positive coefficient, is called *slightly upward*.

Transition probability represents the likelihood that the crude oil WTI price return will stay in the original regime or switch to another regime. According to the matrix of transition probabilities for crude oil WTI (Table 4), if an observation is in regime 1, the following observation has a probability of 97.33% of being in regime 1, a probability of 0.85% of being in regime 2 and a probability of 1.82% of being in regime 3. In other words, if the oil futures market has been observed to have a slightly downward regime a given day, a low negative return is expected to be observed on the following day with a probability of 97.33%. Similarly, the probabilities for regimes 2 and 3 are shown in Table 4. For example, if the market is observed to be in regime 2, the following day is expected to be in regime 2 with a probability of 98.75%. There is also a probability of 1.25% that it will be in regime 1 and essentially 0% probability that it will be in regime 3.

In the six year period studied, the greatest number of observations (728) and the highest probability (0.47) belong to the slightly downward regime (regime 1). The longest duration (80 days) is seen in the slightly upward regime (regime 2); if the returns are positive, the market is expected to stay in the slightly upward regime for 80 days. The smallest number of observations (306), the lowest probability (0.21), and the shortest duration (24) all belong to the sharply downward regime (regime 3). In addition, according to the ergodic probabilities (Table 5), being in regime 1 has the highest probability for any observation at any moment (0.4709).

### 3.2 Markov Switching Autoregressive Model for Crude Oil Brent

The model that best explains the nonlinearity of crude oil Brent is the MSIH(3)-AR(0) which has the smallest values of the HQ (-5.4687) and SIC (-5.4432), and the largest value of the LR statistic (400.2764). According to the LR statistic, the model with the most descriptive power is the MSIH(3)-AR(0) with three regimes.

Coefficients of the MSIH(3)-AR(0) model for crude oil Brent are shown in Table 7. As for the crude oil WTI futures, regime 1 is the slightly downward regime, regime 2 is the slightly upward regime, and regime 3 is the sharply downward regime.

As was the case with the transition possibilities for crude oil WTI, the matrix of transition probabilities for crude oil Brent (Table 8) shows that there is a strong probability that any given observation will be in the same regime as the observation that immediately preceded it. For the models of crude oil Brent futures, the greatest number of observations (782) belong to the slightly downward regime (regime 1). Thus, the probability of the slightly downward regime (0.49) is the highest. The durations of all regimes are very similar, ranging from 25 to 30 days. The smallest number of observations (291) and, the lowest probability (0.20) belong to the sharply downward regime (regime 3) that also has the longest duration (30 days). The ergodic probability of regime 1 (0.4873) is the highest among the three states, a result that is similar to crude oil WTI futures model.

### 3.3 Regime Probabilities and Business Cycle Dates

The regime probabilities of the MSIH(3)-AR(0) model for crude oil futures are shown in Figures 2 and 3. The cycle dates of the models are shown in the tables that follow (Tables 10 and 11). Both the figures and tables indicate that Brent switches more between regime 1 and regime 2 than does WTI. These switching mechanisms cause the difference in the durations noted in Tables 4 and 8. In contrast to regime 1 and regime 2, the regime 3 cycle dates are nearly the same for both the models.

While economic events affect oil market prices, they also affect global demand and supply. Despite the difficulties in specifying the RS dates in detail, we tried to relate the RS mechanism to business cycles on the basis of the Markov Switching time series analysis first applied. If the cycle dates of crude oil futures are considered in relation to the business cycle, the slightly upward regime (regime 2) of our models is associated with the minimum world GDP growth which occurred in 2012 and 2013 (2.5%, and 2.48%, respectively).

Following the steady prices between \$70 and \$80 in the first 11 months of 2010, the last month of 2010 was a growing period which was observed to be in regime 2. The oil futures prices in both the second period of 2014 and the whole of 2015 have followed a falling trend and never reached \$100 USD after July 2014. This period of high volatility has been seen in



regime 1 and regime 3.

## 4 Conclusion

As extreme price movements have recently been recorded, the interest in predicting oil futures prices has been renewed. In this study, we found evidence that the behavior of oil futures prices changes conditional on the state of other related economic variables. The findings indicate that oil prices have a nonlinear pattern involving three regimes. The model which best describes the oil futures logarithmic returns is the MSIH(3)-AR(0). The regime switching mechanism in the MSIH model is specified by the intercept (I) and volatility/heteroskedasticity (H).

The three regimes that the models identify are the same for the two analyzed time series. They are: regime 1, slightly downward (with high volatility); regime 2, slightly upward (with low volatility); and regime 3 sharply downward (with high volatility).

The slightly downward period has the greatest number of observations for both crude oil WTI futures (728 days) and crude oil Brent futures (782 days). On the other hand, the sharply downward period has the smallest number of observations for both crude oil WTI futures (306) and crude oil Brent futures (291). Additionally, the difference in the durations of the periods is important. In particular, the duration of the slightly upward period is significantly different between the crude oil WTI and the crude oil Brent futures, 80 days for crude oil WTI futures and 25 days for crude oil Brent futures. The nonlinear structure of oil futures prices and the probabilities and durations of the regime switching mechanisms in this model are important for international investors who invest in oil futures. For instance, if the market is not too volatile and returns are positive, an investor should expect these returns for 80 days for crude oil WTI futures, and 20 days for crude oil Brent futures. We need to specify that there are many economic events and other reasons which are causes of switching mechanisms, and there is a wide potential study area on this subject. While we consider that volatility or heteroskedasticity (H) is important in specifying the regime switching mechanism in modelling returns, modelling the volatility of oil futures markets with the Markov regime switching GARCH models would be another potential area for study.

## References

Ang, A. and Timmermann, A. (2011). Regime Changes and Financial Markets. Working paper 17182, National Bureau of Economic Research.

- Baum, C. and Zerilli, P. (2016). Jumps and stochastic volatility in crude oil futures prices using conditional moments of integrated volatility. *Energy Economics*, 53(C):175–181.
- Bernard, J.-T., Khalaf, L., Kichian, M., and McMahon, S. (2015). The Convenience Yield and the Informational Content of the Oil Futures Price. *The Energy Journal*, 36(2):39–46.
- Bildirici, M. E., Aykaç Alp, E., Ersin, Ö. Ö., and Bozoklu, Ü. (2010). *İktisatta Kullanılan Doğrusal Olmayan Zaman Serisi Yöntemleri*.
- Breeden, D. T. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics*, 7(3):265–296.
- Breeden, D. T. (1980). Consumption Risk in Futures Markets. *The Journal of Finance*, 35(2):503–520.
- Breeden, D. T. and Litzenberger, R. H. (1978). Prices of State-contingent Claims Implicit in Option Prices. *The Journal of Business*, 51(4):621–51.
- Chevallier, J. (2013). Price relationships in crude oil futures: new evidence from CFTC disaggregated data. *Environmental Economics and Policy Studies*, 15(2):133–170.
- Dusak, K. (1973). Futures Trading and Investor Returns: An Investigation of Commodity Market Risk Premiums. *Journal of Political Economy*, 81(6):1387–1406.
- Elder, J., Miao, H., and Ramchander, S. (2014). Price discovery in crude oil futures. *Energy Economics*, 46(S1):S18–S27.
- Fong, W. M. and See, K. H. (2002). A Markov switching model of the conditional volatility of crude oil futures prices. *Energy Economics*, 24(1):71–95.
- Franses, P. H. and van Dijk, D. (2000). *Non-Linear Time Series Models in Empirical Finance*. Cambridge University Press.
- Guidolin, M. and Timmermann, A. (2006). An econometric model of nonlinear dynamics in the joint distribution of stock and bond returns. *Journal of Applied Econometrics*, 21(1):1–22.
- Hamilton, J. D. (1989). A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle. *Econometrica*, 57(2):357–384.
- Kordnoori, S., Mostafaei, H., and Ostadrahimi, M. (2013). Modelling the fluctuations of Brent oil prices by a probabilistic Markov chain. *African Journal of Business Management*, 7(17):1648–1654.

- Koy, A. (2017). International Credit Default Swaps Market During European Crisis: A Markov Switching Approach. In *Global Financial Crisis and Its Ramifications on Capital Markets*, pages 431–443. Springer.
- Krolzig, H.-M. (2000). Predicting Markov-switching vector autoregressive processes. Department of Economics Working Papers 2000-W31, Nuffield College, University of Oxford.
- Markov, N. (2010). A Regime Switching Model for the European Central Bank. Research Papers by the Institute of Economics and Econometrics 10091, Geneva School of Economics and Management, University of Geneva.
- Medhioub, I. (2015). A Markov Switching Three Regime Model of Tunisian Business Cycle. *American Journal of Economics*, 5(3):394–403.
- Perez-Quiros, G. and Timmermann, A. (2001). Business cycle asymmetries in stock returns: Evidence from higher order moments and conditional densities. *Journal of Econometrics*, 103(1-2):259–306. Studies in estimation and testing.
- Rockwell, C. S. et al. (1967). Normal backwardation, forecasting, and the returns to commodity futures traders. *Food Research Institute Studies*, 7(1967):107–130.
- Schwartz, E. and Smith, J. E. (2000). Short-Term Variations and Long-Term Dynamics in Commodity Prices. *Management Science*, 46(7):893–911.
- Vo, M. T. (2009). Regime-switching stochastic volatility: Evidence from the crude oil market. *Energy Economics*, 31(5):779–788.
- Working, H. (1949). The theory of price of storage. *The American Economic Review*, 39(6):1254–1262.
- Zhang, Y.-J. and Zhang, L. (2015). Interpreting the crude oil price movements: Evidence from the Markov regime switching model. *Applied Energy*, 143(C):96–109.
- Zlatcu, I., Kubinski, M., and Barnea, D. (2015). Fuel Price Volatility and Asymmetric Transmission of Crude Oil Price Changes to Fuel Prices. *Theoretical and Applied Economics*, 22(4):33–44.

Figure 1: Crude Oil Futures Prices, Crude Oil Brent Futures and Differential Natural Logarithmic Prices

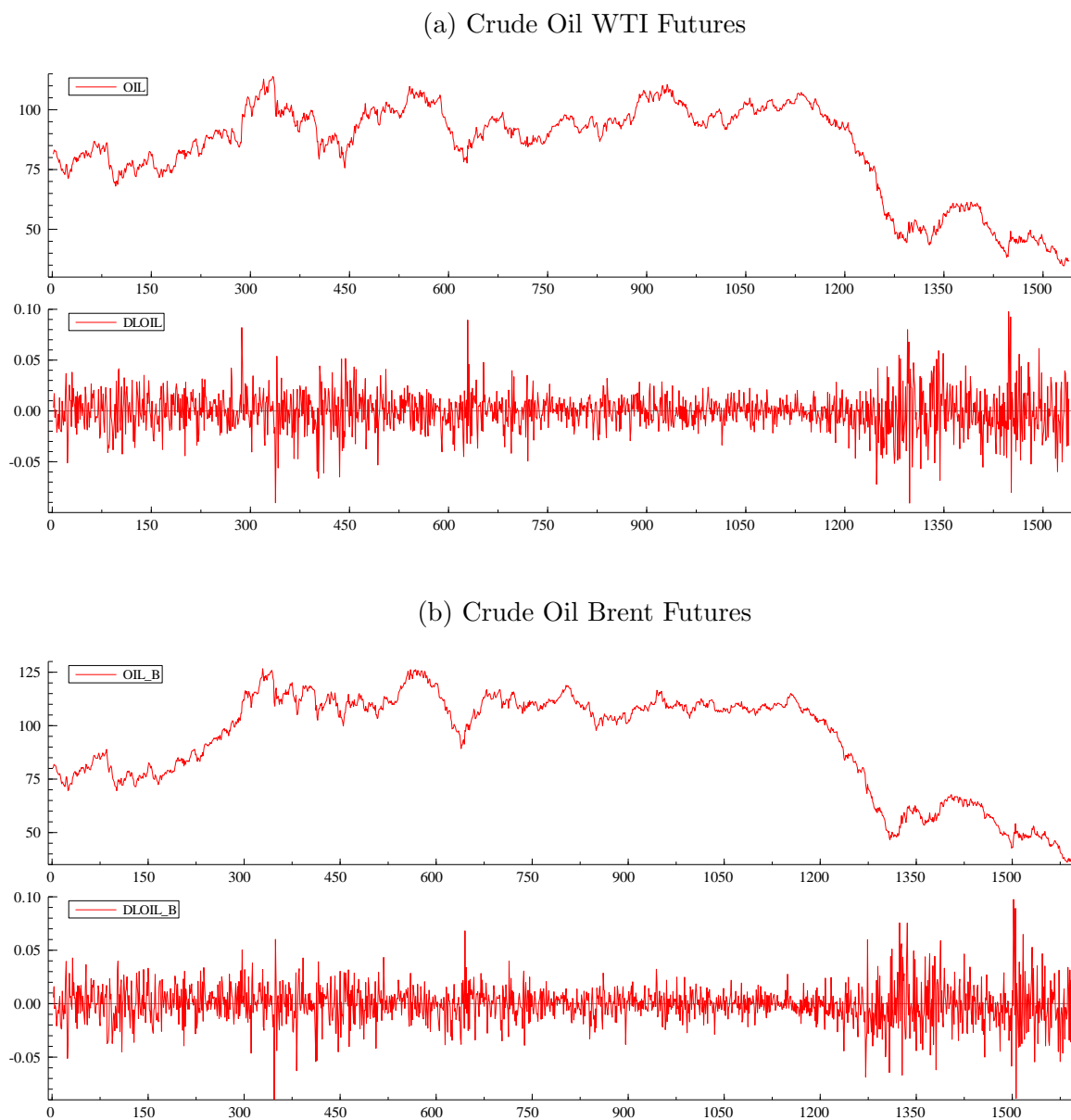


Figure 2: Regime Probabilities – Crude Oil WTI

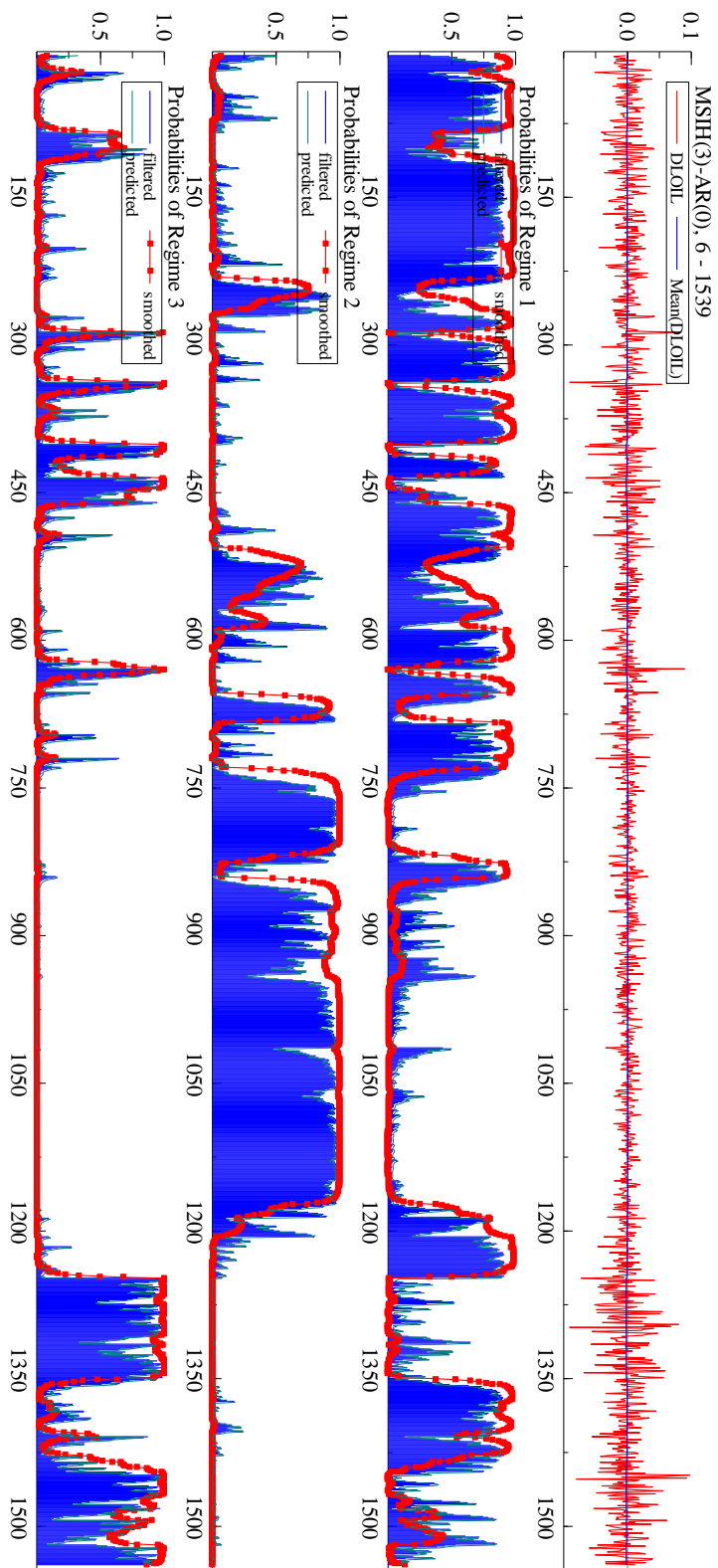


Figure 3: Regime Probabilities – Crude Oil Brent

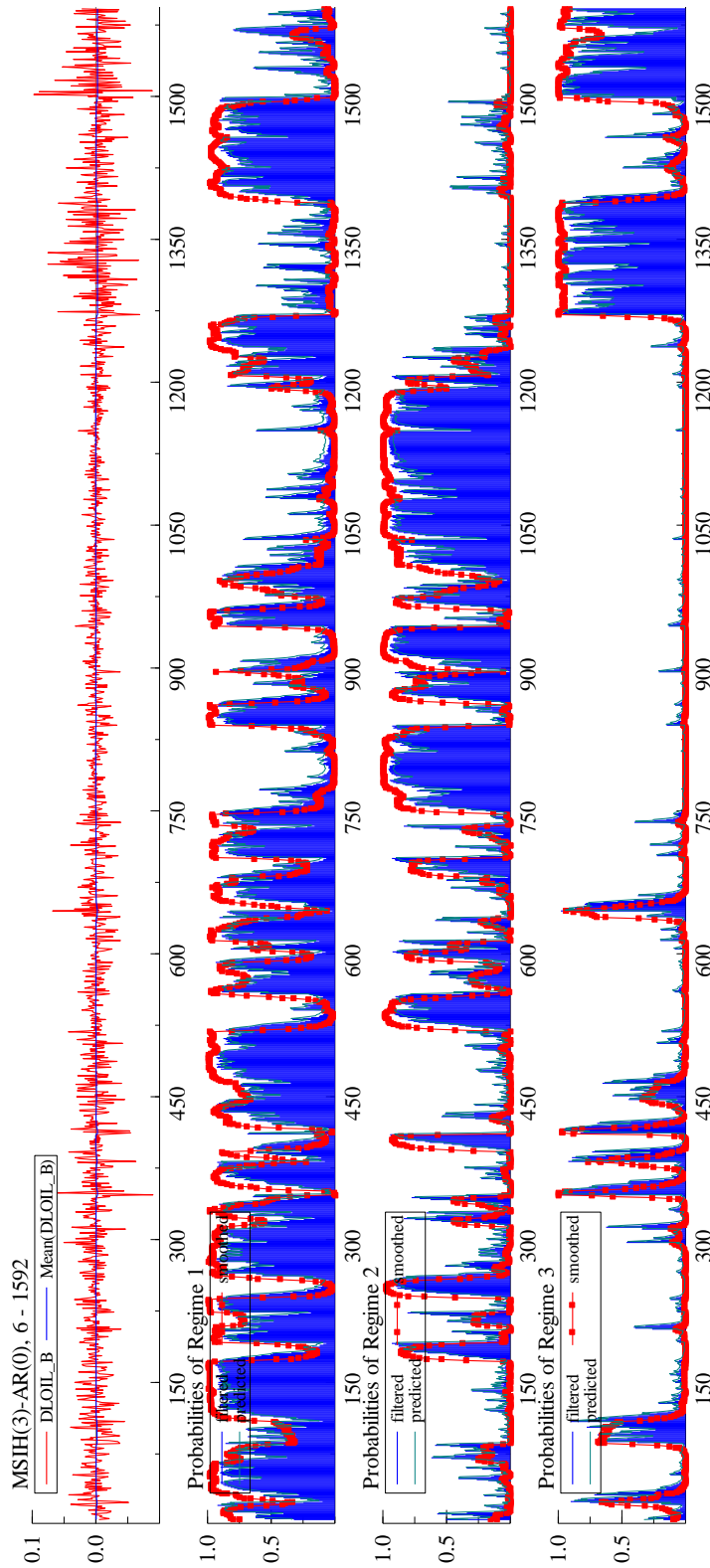


Table 1: Descriptive Statistics of Analyzed Variables

	<b>WTI</b>	<b>BRENT</b>
<b>Mean</b>	84.49617	94.91823
<b>Median</b>	90.80000	105.5400
<b>Maximum</b>	113.9300	126.6500
<b>Minimum</b>	34.73000	42.69000
<b>Std. Dev</b>	19.19545	21.55654
<b>Skewness</b>	-0.977078	-0.785413
<b>Kurtosis</b>	2.872541	2.333900
<b>Jarque-Bera</b>	245.9176	186.6795
<b>Probability</b>	0.000000	0.000000
<b>Sum</b>	130039.6	146079.2
<b>Sum Sq. Dev.</b>	566699.5	714684.8

Table 2: Information Criterion Values for Crude Oil WTI

<b>Model</b>	<b>log-likelihood</b>	<b>AIC</b>	<b>HQ</b>	<b>SIC</b>	<b>LR linearity test</b>	<b>p-value of the Davies' test</b>
MSI(3)-AR(0)	3991.8353	-5.1779	-5.1650	-5.1432	203.7858	0.0000
MSIH(2)-AR(0)	4045.4444	-5.2529	-5.2451	-5.2320	311.0041	0.0000
MSI(3)-AR(0)	4063.3734	-5.2821	-5.2666	-5.2404	370.1262	0.0000
MSI(3)-AR(1)	3991.1921	-5.1792	-5.1650	-5.1410	202.6508	0.0000
MSIH(2)-AR(1)	4043.0675	-5.2519	-5.2428	-5.2276	306.4016	0.0000
MSIH(3)-AR(1)	4073.5898	-5.2838	-5.2670	-5.2386	367.4462	0.0000
MSI(3)-AR(2)	3982.2975	-5.1764	-5.1609	-5.1347	201.8906	0.0000
MSIH(2)-AR(2)	4033.9452	-5.2490	-5.2386	-5.2211	305.1860	0.0000
MSIH(3)-AR(2)	4071.0222	-5.2826	-5.2645	-5.2339	367.2806	0.0000
MSI(3)-AR(3)	3982.3519	-5.1752	-5.1583	-5.1299	201.9004	0.0000
MSIH(2)-AR(3)	4034.0096	-5.2477	-5.2361	-5.2164	305.2158	0.0000
MSIH(3)-AR(3)	4067.9061	-5.2807	-5.2613	-5.2285	366.9508	0.0000
MSI(3)-AR(4)	3982.6001	-5.1742	-5.1561	-5.1255	202.3419	0.0000
MSIH(2)-AR(4)	4034.0103	-5.2464	-5.2335	-5.2116	305.1623	0.0000
MSIH(3)-AR(4)	4064.7701	-5.2787	-5.2580	-5.2231	366.6819	0.0000

Table 3: Coefficients for Models of Crude Oil WTI

<b>MSIH(3)-AR(0)</b>			
	<b>Coefficient</b>	<b>Standard Error</b>	<b>t-value</b>
<b>Constant (Regime 1)</b>	-0.0005	0.0007	-0.7295
<b>Constant (Regime 2)</b>	0.0003	0.0005	0.6868
<b>Constant (Regime 3)</b>	-0.0019	0.0019	-0.9940
<b>Standard Error (Regime 1)</b>	0.0169		
<b>Standard Error (Regime 2)</b>	0.0104		
<b>Standard Error (Regime 3)</b>	0.0317		

Table 4: Matrix of Transition Probabilities for Crude Oil WTI

<b>Model</b>	<b>Regime</b>	<b>Regime 1</b>	<b>Regime 2</b>	<b>Regime 3</b>
<b>MSIH(3)-AR(0)</b>	<b>Regime 1</b>	0.9733	0.0085	0.0182
	<b>Regime 2</b>	0.0125	0.9875	0.0000
	<b>Regime 3</b>	0.0408	0.0000	0.9591

Table 5: Number of Observations for Crude Oil WTI

<b>Model</b>	<b>Regime</b>	<b>Number of observations</b>	<b>Probability</b>	<b>Duration (days)</b>
<b>MSIH(3)-AR(0)</b>	<b>1</b>	727.8	0.4709	37.51
	<b>2</b>	499.9	0.3197	79.74
	<b>3</b>	306.4	0.2094	24.46



Table 6: Information Criterion Values for Crude Oil Brent

Model	log-likelihood	AIC	HQ	SIC	LR linearity test	p-value of the Davies' test
MSIH(2)-AR(0)	4328.0040	-5.4468	-5.4392	-5.4265		0,0000
MSIH(3)-AR(0)	4374.2848	-5.4837	-5.4687	-5.4432	400.2764	0,0000
MSI(3)-AR(1)	4267.6265	-5.3644	-5.3505	-5.3272	204.0874	0,0000
MSIH(2)-AR(1)	4330.4120	-5.4485	-5.4397	-5.4248	329.6585	0,0000
MSIH(3)-AR(1)	4373.5604	-5.4850	-5.4687	-5.4411	398.1731	0,0000
MSI(3)-AR(2)	4267.6750	-5.3632	-5.3481	-5.3226	203.6919	0,0000
MSIH(2)-AR(2)	4330.7581	-5.4477	-5.4377	-5.4206	329.8582	0,0000
MSIH(3)-AR(2)	4371.2413	-5.4843	-5.4667	-5.4369	398.3565	0,0000
MSI(3)-AR(3)	4267.7990	-5.3621	-5.3457	-5.3181	202.9672	0,0000
MSIH(2)-AR(3)	4330.7605	-5.4465	-5.4351	-5.4160	328.8901	0,0000
MSIH(3)-AR(3)	4367.9474	-5.4823	-5.4635	-5.4316	397.0194	0,0000
MSI(3)-AR(4)	4268.7489	-5.3620	-5.3444	-5.3146	204.3794	0,0000
MSIH(2)-AR(4)	4030.7984	-5.4452	-5.4327	-5.4114	328.4784	0,0000
MSIH(3)-AR(4)	4364.8448	-5.4806	-5.4605	-5.4264	396.5712	0,0000

Table 7: Coefficients for Models of Crude Oil Brent

<b>MSIH(3)-AR(0)</b>			
	<b>Coefficient</b>	<b>Standard Error</b>	<b>t-value</b>
<b>Constant (Regime 1)</b>	-0.0003	0.0007	-0.3893
<b>Constant (Regime 2)</b>	0.0003	0.0005	0.7071
<b>Constant (Regime 3)</b>	-0.0025	0.0018	-1.3949
<b>Standard Error (Regime 1)</b>	0.0160		
<b>Standard Error (Regime 2)</b>	0.0085		
<b>Standard Error (Regime 3)</b>	0.0293		

Table 8: Matrix of Transition Probabilities - Crude Oil Brent

Model	Regime	Regime 1	Regime 2	Regime 3
MSIH(3)-AR(0)	Regime 1	0.9629	0.0262	0.0109
	Regime 2	0.0363	0.9595	0.0042
	Regime 3	0.0335	0.0002	0.9663

Table 9: Number of Observations for Crude Oil Brent

Model	Regime	Number of observations	Probability	Duration (days)
MSIH(3)-AR(0)	1	781.5	0.4873	26.97
	2	518.1	0.3160	24.68
	3	291.4	0.1967	29.68

Table 10: Cycle Dates - Crude Oil WTI

Regime 1		Regime 2		Regime 3	
Date	Probability	Date	Probability	Date	Probability
11/01/2010 - 03/05/2010	[0.9087]	07/12/2010 - 31/12/2010	[0.6927]	04/05/2010 - 03/06/2010	[0.6017]
26/05/2010 - 06/12/2010	[0.9515]	17/01/2012 - 17/02/2012	[0.6258]	18/02/2011 - 23/02/2011	[0.7989]
03/01/2011 - 17/02/2011	[0.8146]	08/08/2012 - 13/09/2012	[0.8238]	04/05/2011 - 13/05/2011	[0.8618]
24/02/2011 - 03/05/2011	[0.9017]	26/11/2012 - 02/04/2013	[0.9497]	03/08/2011 - 22/08/2011	[0.8954]
16/05/2011 - 02/08/2011	[0.9002]	07/05/2013 - 18/08/2014	[0.9673]	19/09/2011 - 27/10/2011	[0.8197]
23/08/2011 - 16/09/2011	[0.7674]			21/06/2012 - 09/07/2012	[0.7755]
28/10/2011 - 13/01/2012	[0.8911]			25/11/2014 - 16/04/2015	[0.9660]
21/02/2012 - 20/06/2012	[0.7842]			07/08/2015 - 31/12/2015	[0.8432]
10/07/2012 - 07/08/2012	[0.8583]				
14/09/2012 - 23/11/2012	[0.9068]				
03/04/2013 - 06/05/2013	[0.8106]				
19/08/2014 - 24/11/2014	[0.8538]				
17/04/2015 - 06/08/2015	[0.8590]				

Table 11: Cycle Dates - Crude Oil Brent

Regime 1		Regime 2		Regime 3	
Date	Probability	Date	Probability	Date	Probability
11/01/2010 - 01/02/2010	[0.7954]	03/09/2010 - 27/09/2010	[0.7484]	02/02/2010 - 08/02/2010	[0.5973]
28/01/2010 - 03/05/2010	[0.8611]	06/12/2010 - 30/12/2010	[0.8899]	04/05/2010 - 04/06/2010	[0.6180]
07/06/2010 - 02/09/2010	[0.9535]	12/07/2011 - 02/08/2011	[0.8283]	03/05/2011 - 12/05/2011	[0.8499]
28/09/2010 - 03/12/2010	[0.8792]	06/01/2012 - 22/02/2012	[0.8962]	21/06/2011 - 28/06/2011	[0.6228]
31/12/2010 - 02/05/2011	[0.8515]	18/04/2012 - 02/05/2012	[0.7293]	03/08/2011 - 11/08/2011	[0.8642]
13/05/2011 - 20/06/2011	[0.8286]	21/08/2012 - 13/09/2012	[0.7152]	20/06/2012 - 06/07/2012	[0.7296]
29/06/2011 - 11/07/2011	[0.7349]	22/11/2012 - 01/04/2013	[0.9215]	26/11/2014 - 17/04/2015	[0.9685]
12/08/2011 - 05/01/2012	[0.8812]	07/05/2013 - 17/06/2013	[0.7661]	19/08/2015 - 31/12/2015	[0.9184]
23/02/2012 - 17/04/2012	[0.7995]	25/06/2013 - 23/08/2013	[0.9151]		
03/05/2012 - 19/06/2012	[0.7854]	23/09/2013 - 16/10/2013	[0.7434]		
09/07/2012 - 20/08/2012	[0.8487]	13/11/2013 - 29/08/2014	[0.9271]		
14/09/2012 - 21/11/2012	[0.8750]				
02/04/2013 - 06/05/2013	[0.9375]				
18/06/2013 - 24/06/2013	[0.6836]				
14/08/2013 - 20/09/2013	[0.9073]				
17/10/2013 - 12/11/2013	[0.7327]				
01/09/2014 - 25/11/2014	[0.8134]				
20/04/2015 - 18/08/2015	[0.9052]				